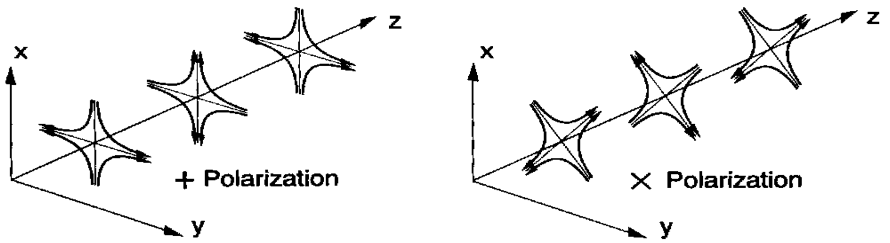
Gravitational Waves (GW)

- Emission: **Accelerated quadrupole bulk mass-energy motion.**

Quadrupole approximation:

$$h_{jk}^{TT}(t,\vec{x}) = \left[\frac{2}{c^4}\frac{G}{|\vec{x}|}\ddot{I}_{jk}(t-\frac{|\vec{x}|}{c})\right]^{TT}$$

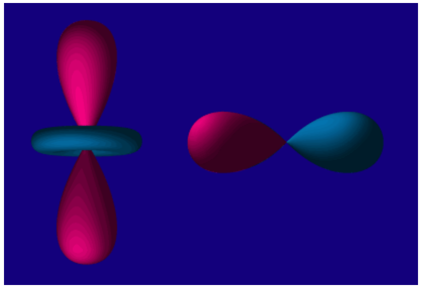
Details: See Jay Marx's and Alan Weinstein's lectures tomorrow!



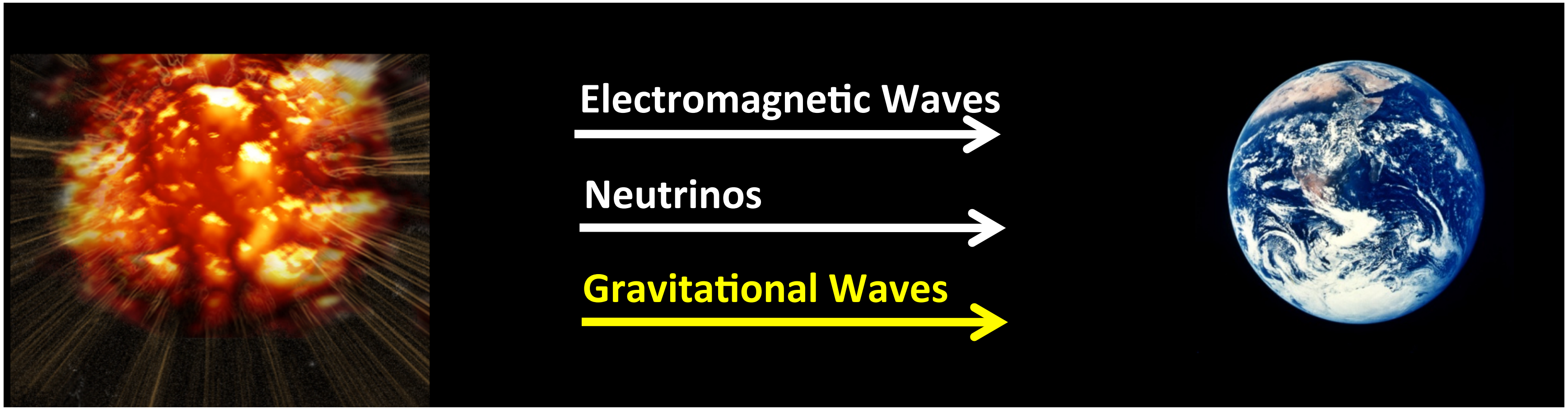
$$\frac{G}{c^4} \approx 10^{-49}\,\mathrm{s^2\,g^{-1}\,cm^{-1}}$$

$$10\,\mathrm{kpc} \approx 3\times10^{22}\,\mathrm{cm}$$

-> For astrophysical sources, must measure relative displacements of < 10^{-22}



The Sound of Cosmic Explosions?



Electromagnetic Waves

Neutrinos

Gravitational Waves

- Time-changing aspherical mass-energy flux -> Gravitational Waves
- Some back of the envelope physics:
  We can estimate the frequency at which GWs are emitted!

Free fall time of a self-gravitating system

$$\tau_{\mathrm{ff}} = \frac{1}{4}\sqrt{\frac{3\pi}{2G\bar{\rho}}} \approx 0.54\frac{1}{\sqrt{G\bar{\rho}}} \qquad f_{\mathrm{GW}} \approx 1/\tau_{\mathrm{ff}} \approx 2\sqrt{G\bar{\rho}}$$

- Example: Volumetric mean density of the inner supernova core around bounce is 10^{13} g/cm^3 -> f_{GW} will be of order 1000 Hz!

Gravitational-Waves from Core-Collapse Supernovae

Recent reviews: Ott '09, Kotake '11, Fryer & New '11

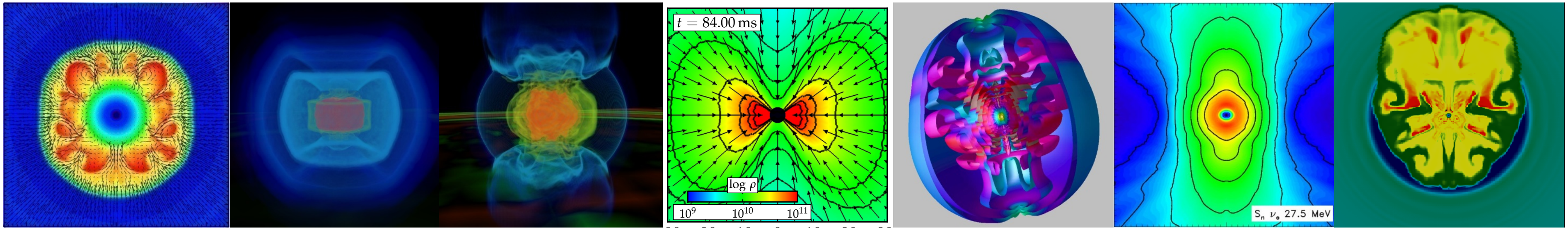
Need:

$$h_{jk}^{TT}(t,\vec{x}) = \left[\frac{2}{c^4}\frac{G}{|\vec{x}|}\ddot{I}_{jk}\left(t-\frac{|\vec{x}|}{c}\right)\right]^{TT}$$

→ accelerated aspherical (quadrupolar) mass-energy motions

Candidate Emission Processes:

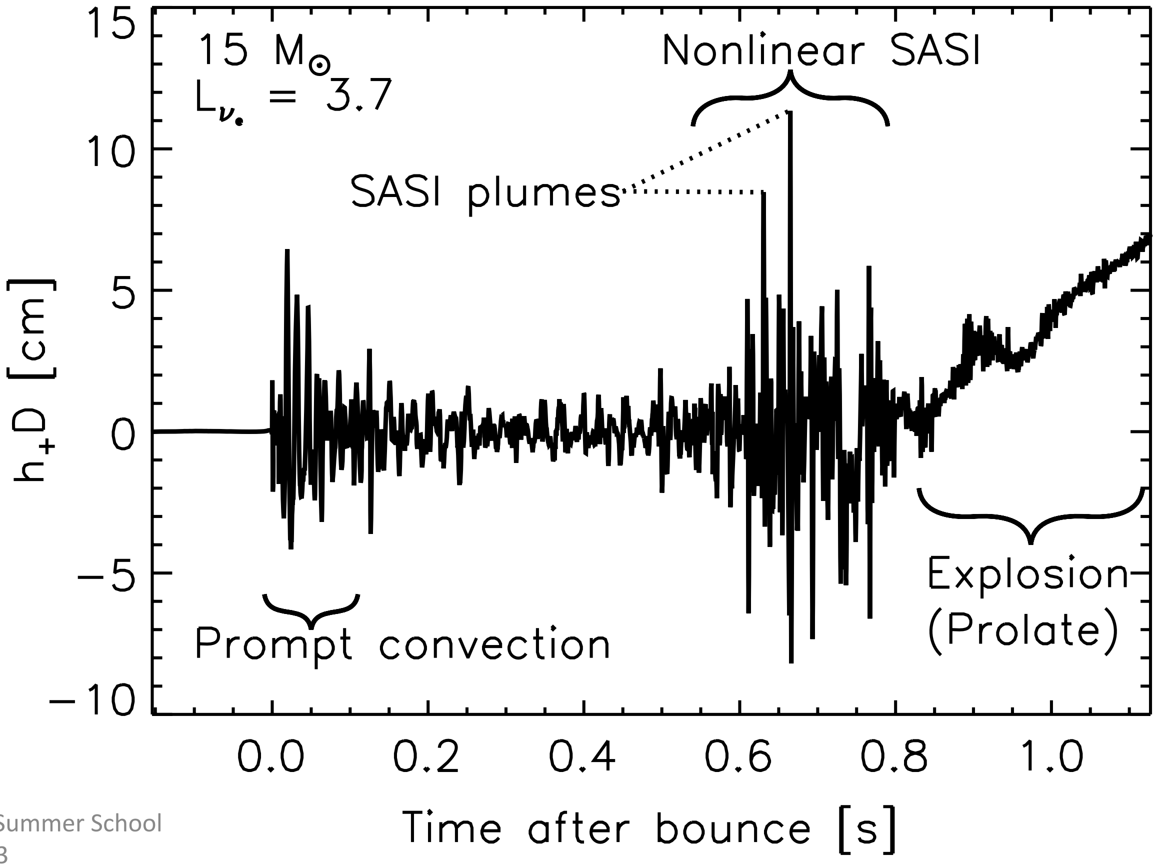
- Convection and SASI
- Rotating collapse & bounce
- Rotational 3D instabilities
- Black hole formation
- Pulsations of the protoneutron star
- Anisotropic neutrino emission
- Aspherical accelerated outflows
- Magnetic stresses



GWs from Convection & SASI

Recent work: Kotake+ ‘09, ‘11, Murphy+’09, Yakunin+’10 E. Müller+’12, B.Müller+‘13

- Prompt convection soon after bounce (Marek+ ‘09, Ott ‘09).
- Neutrino-driven convection & SASI (recent: Murphy+’09, Yakunin+10, Müller+12).
- Protoneutron star convection (e.g., Keil+ ’96, Müller+’04)

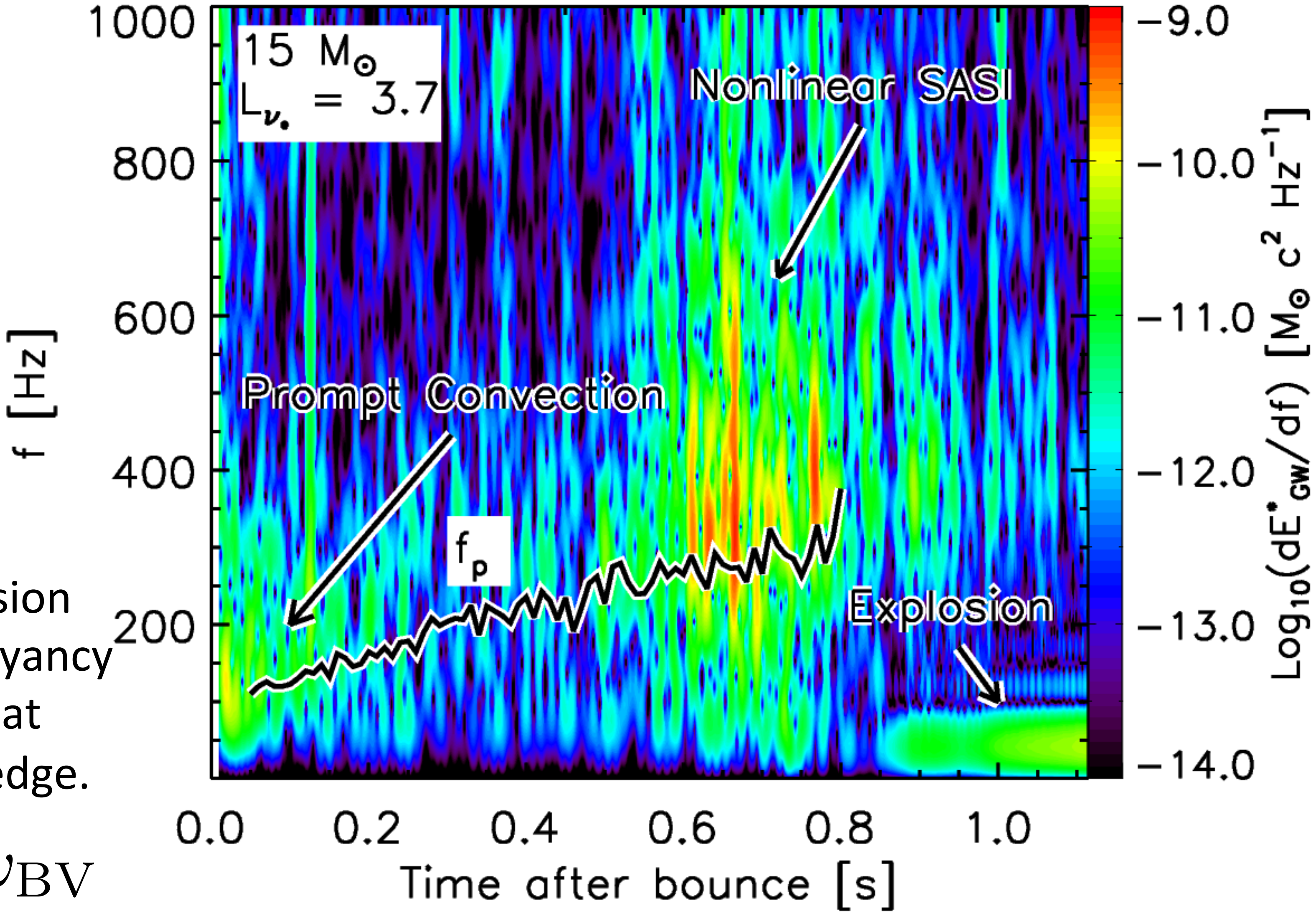


Murphy+ ’09, using simplified heating/cooling scheme.

Expect also: Correlations with neutrino signal. Lund+ ’10,’12, Marek+’09, Brandt+’11

Time-Frequency Analysis of GWs

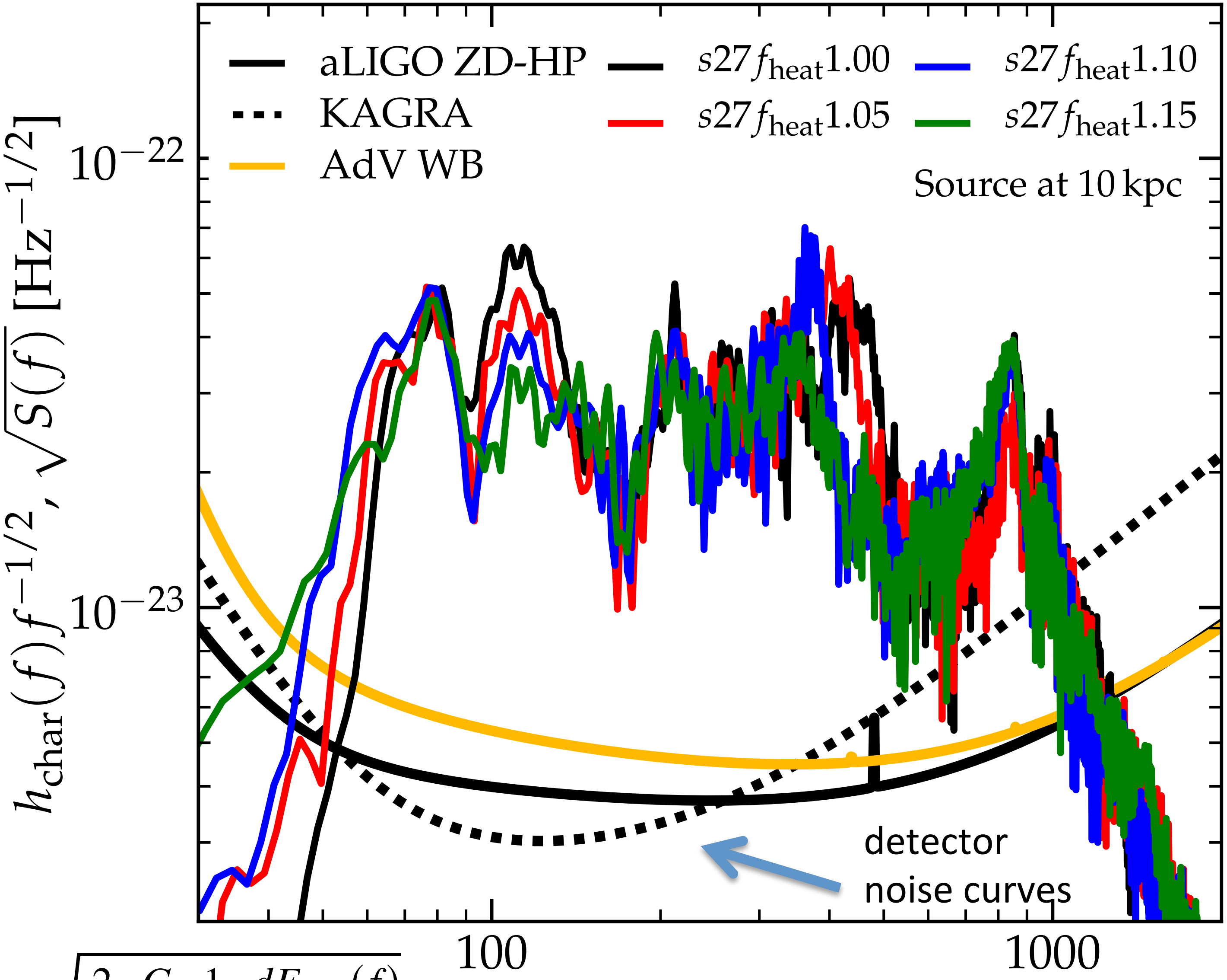
Murphy, Ott, Burrows '09, see also B. Müller+'13



Peak emission traces buoyancy frequency at proto-NS edge.

$$f_p \sim \frac{\omega_{\rm BV}}{2\pi}$$

Can we observe GWs from Core-Collapse Supernovae?



Ott+13,
ApJ 768:115

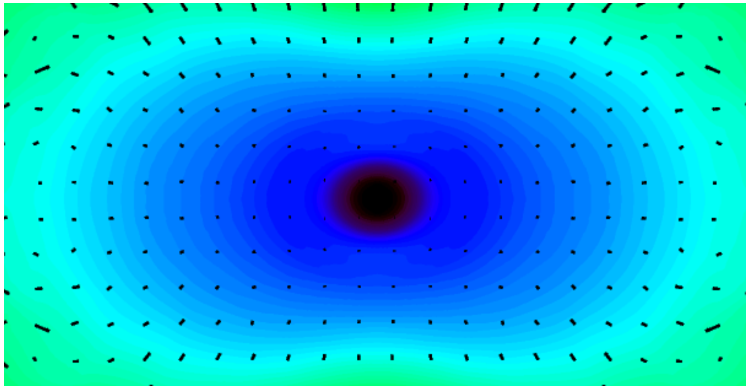
$$h_{\rm char}(f) = \sqrt{\frac{2}{\pi^2}\frac{G}{c^3}\frac{1}{D^2}\frac{dE_{\rm GW}(f)}{df}},$$

$$E_{\rm GW} \gtrsim 4 \times 10^{-10}\, M_\odot c^2$$

GWs from Rotating Collapse & Bounce

Recent work: Dimmelmeier+ ’08, Scheidegger+ ‘10, Ott+ ’12, Kuroda+ ‘13

Rapid rotation:
Oblate deformation of the inner core



- Most extensively studied GW emission in core collapse
- **Axisymmetric: ONLY h_+**
- Simplest GW emission process: **Rotation** + **Gravity** + **Stiffening of nuclear EOS**.
- Strong signals for rapid rotation (-> millisecond proto-NS).

GWs from Rotating Collapse & Bounce

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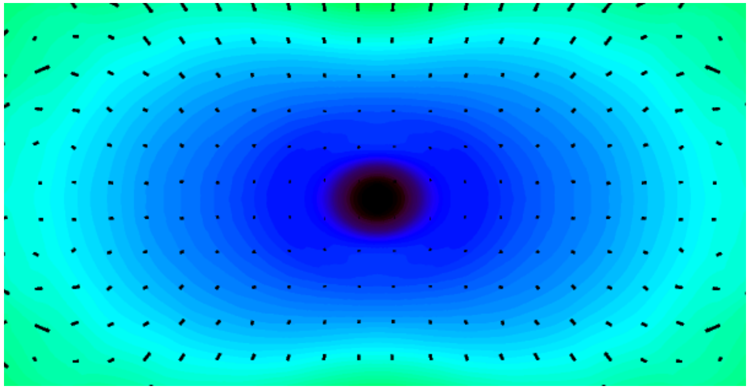

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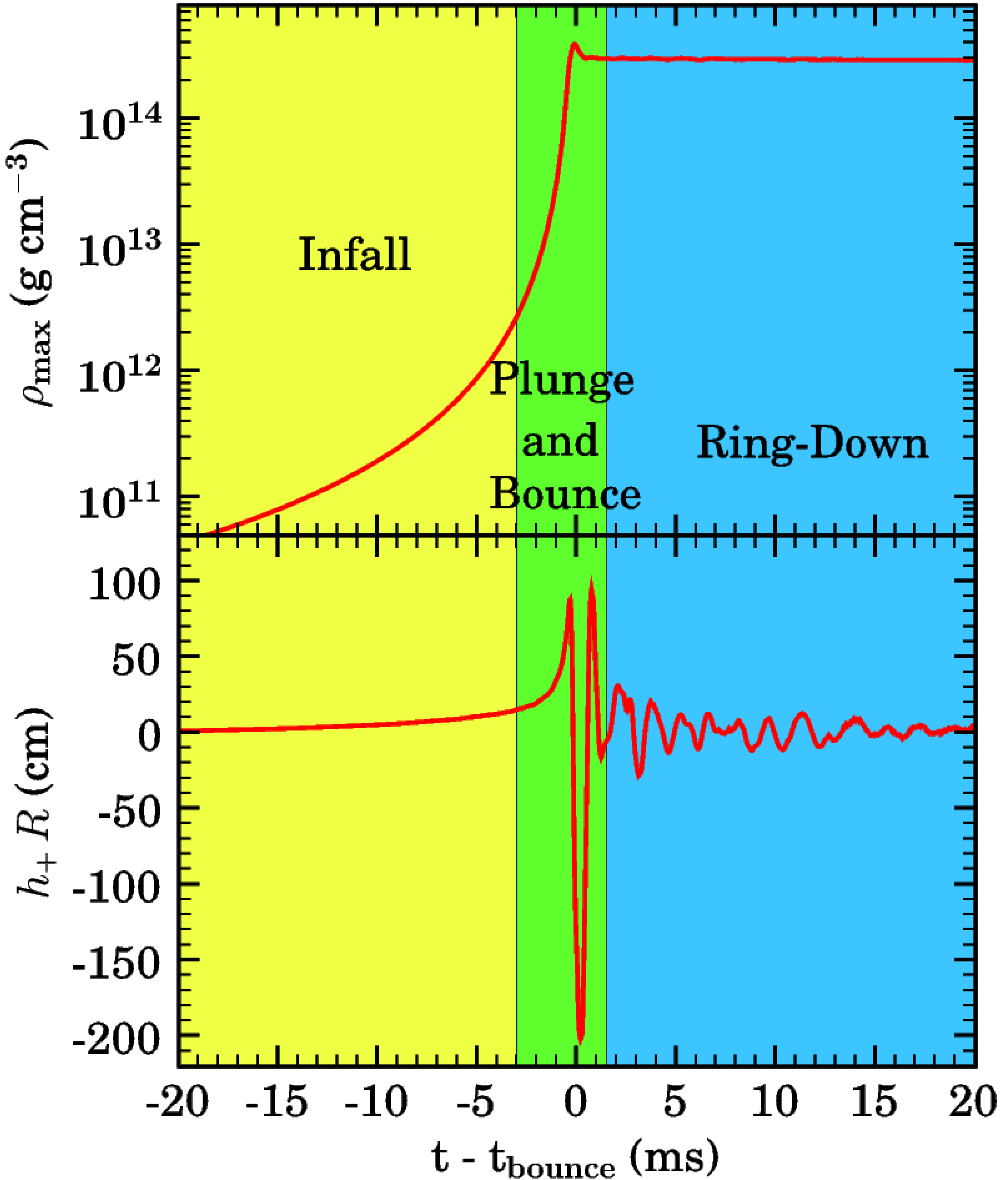
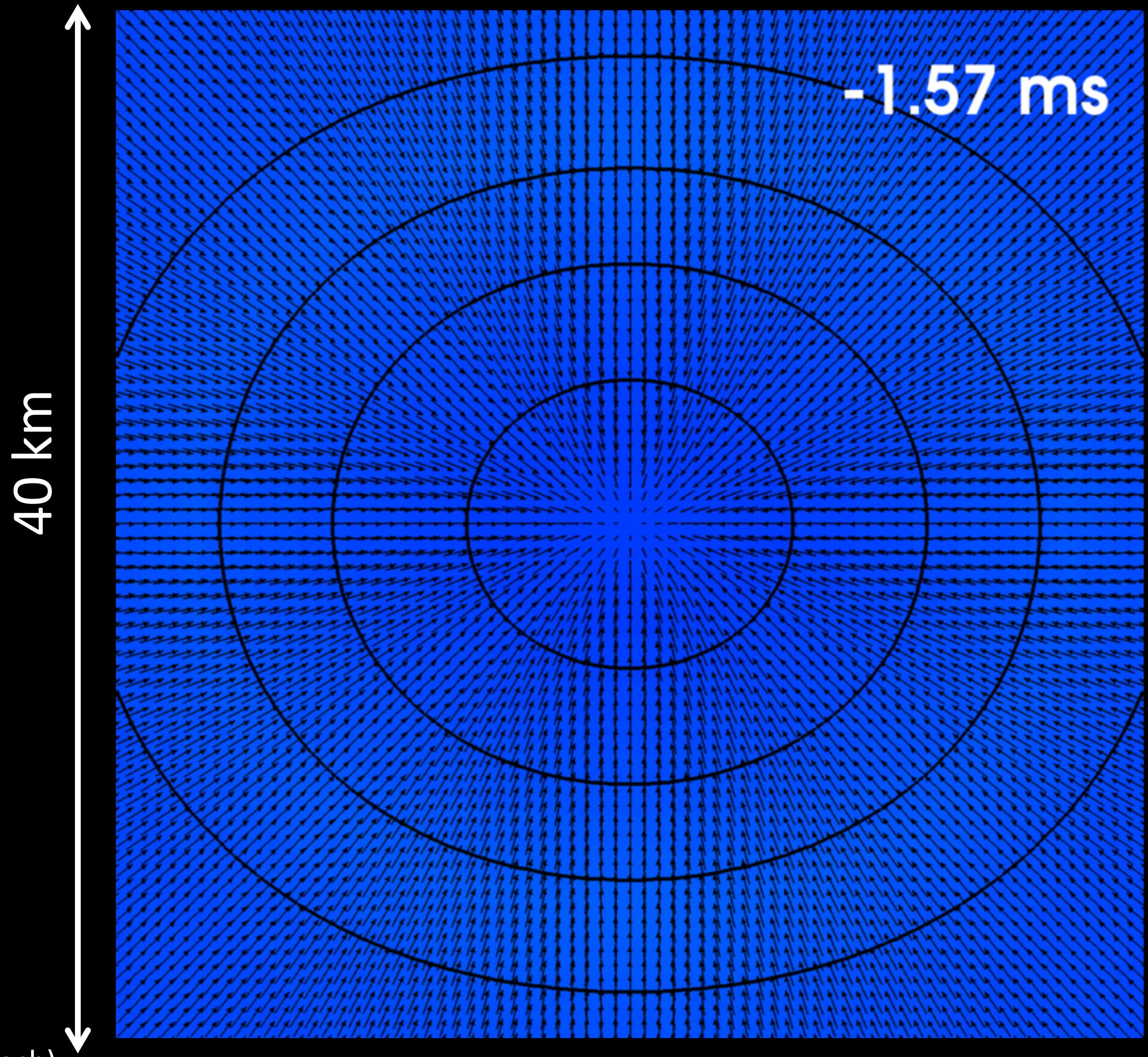


Ott+12

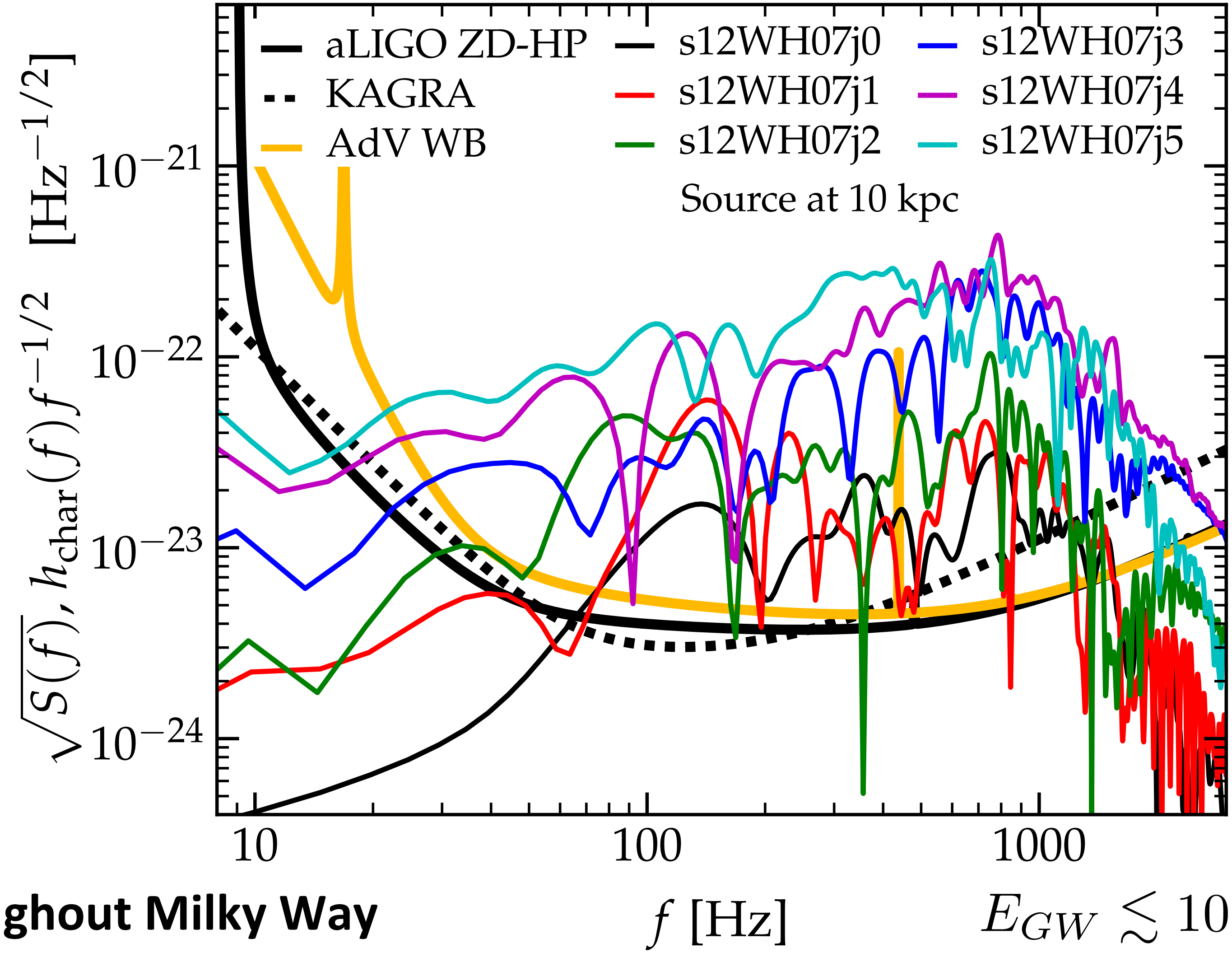


Movie by
Steve Drasco
(Grinnell/Caltech)

Can we observe this?

Ott+ '12, PRD

Gravitational Waves

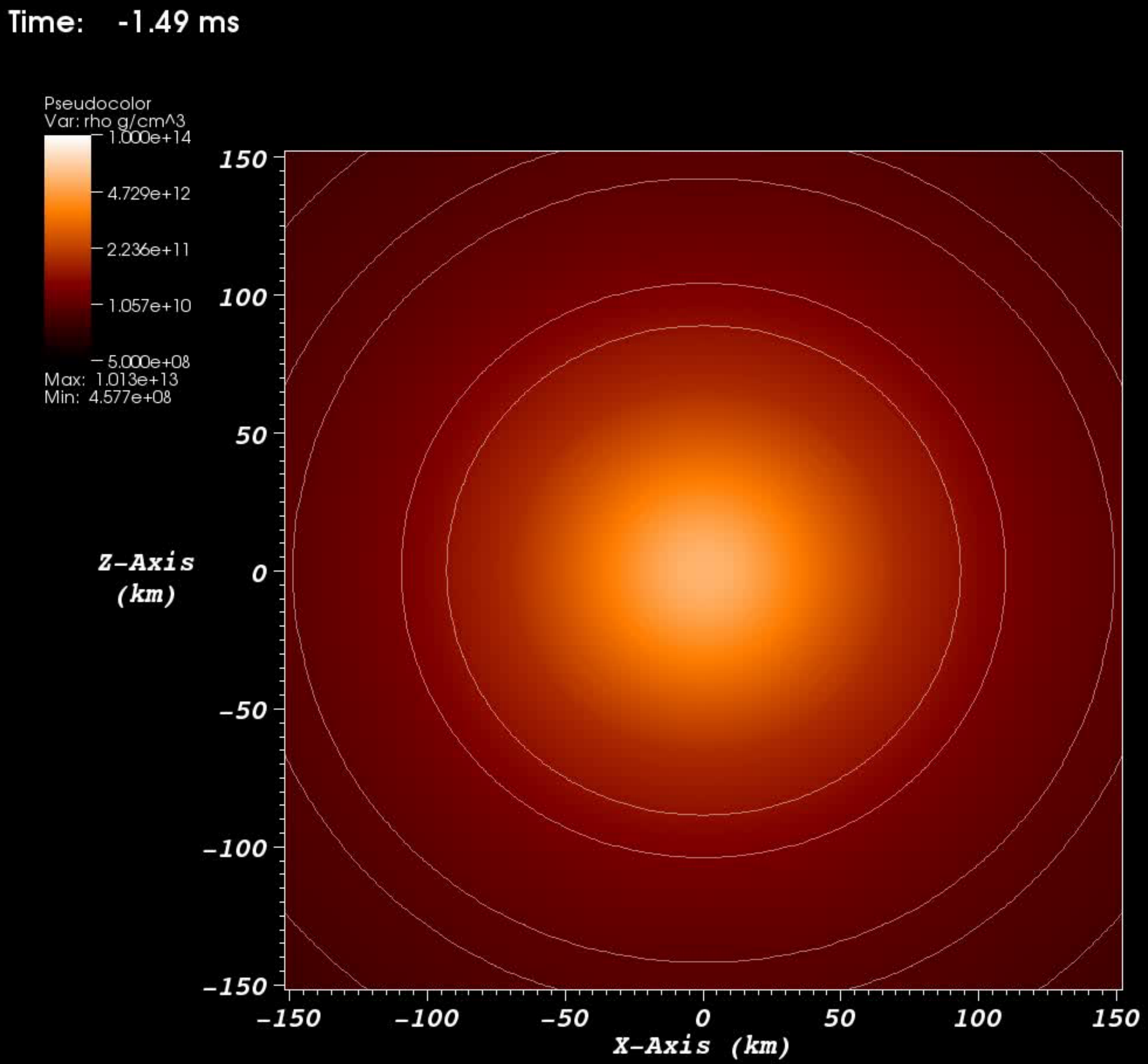


-> **Throughout Milky Way with aLIGO**

$E_{GW} \lesssim 10^{-8} M_\odot c^2$

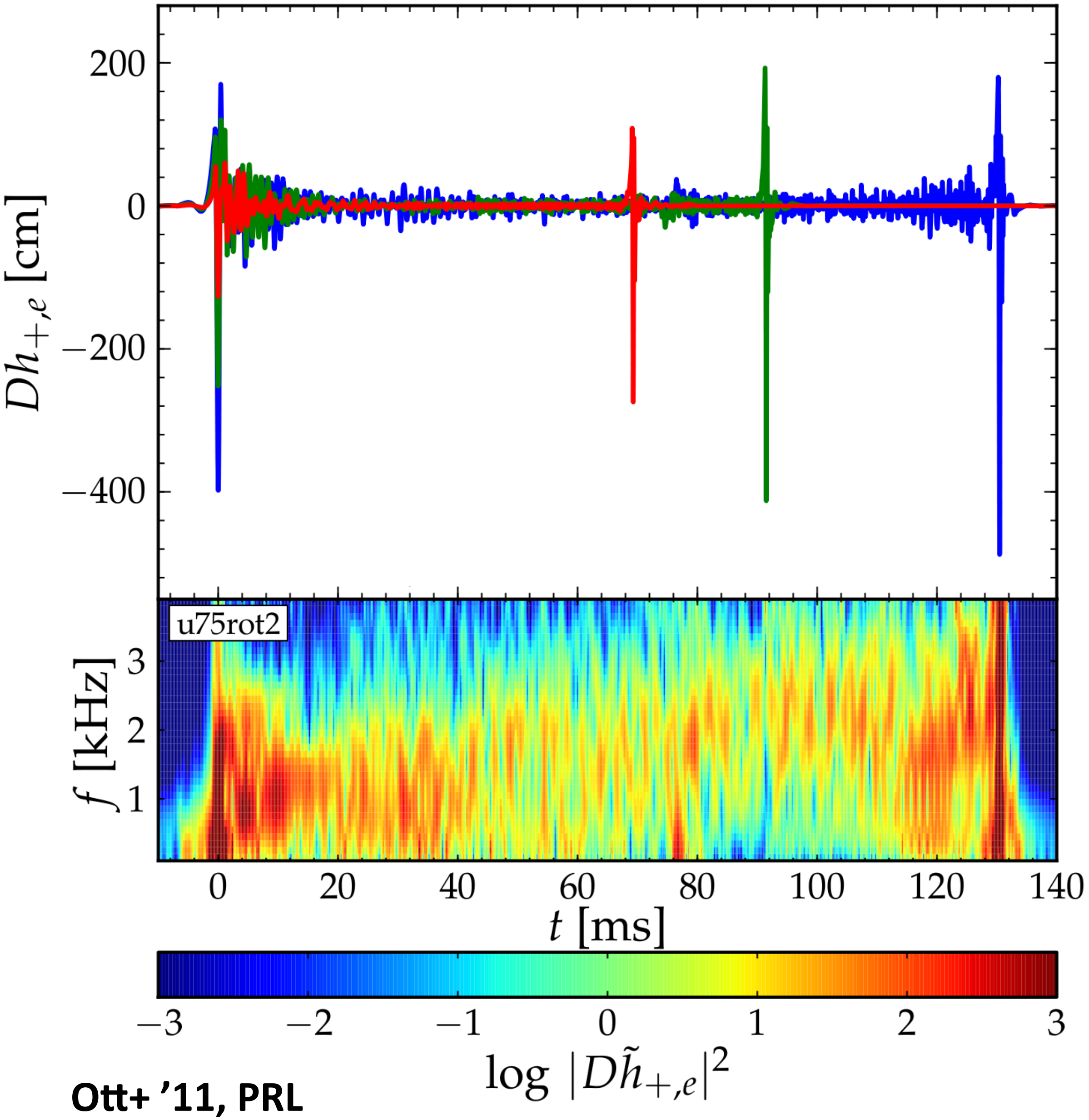
3+1 GR simulation, simplified microphysics

u75 progenitor of Woosley+02



Ott+11

Gravitational Waves from BH Formation



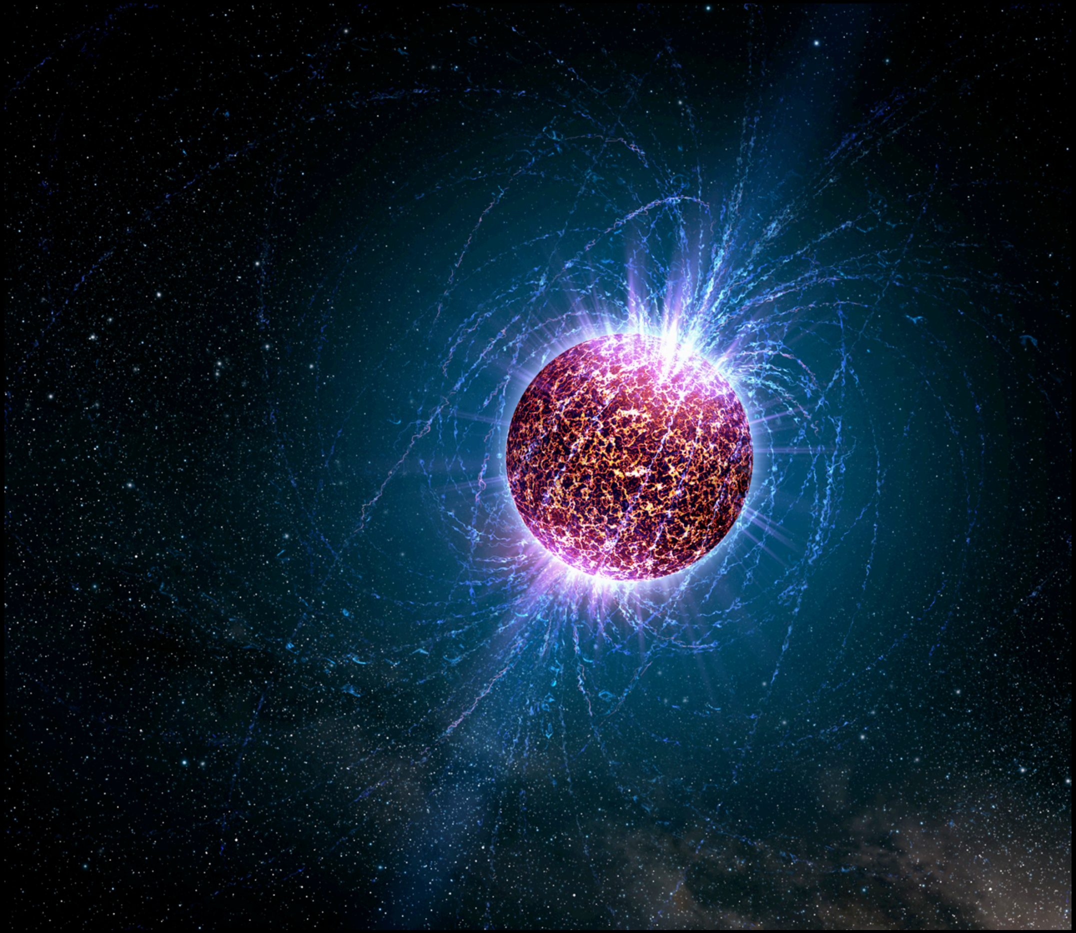
Rotating Black Hole Formation

$$E_{\mathrm{GW}} \sim 10^{-7}\,M_\odot\,c^2$$

But: observable only for nearby (galaxy/SMC/ LMC events)

Ott+ ’11, PRL

Neutron Stars & Constraints on the Nuclear Equation of State

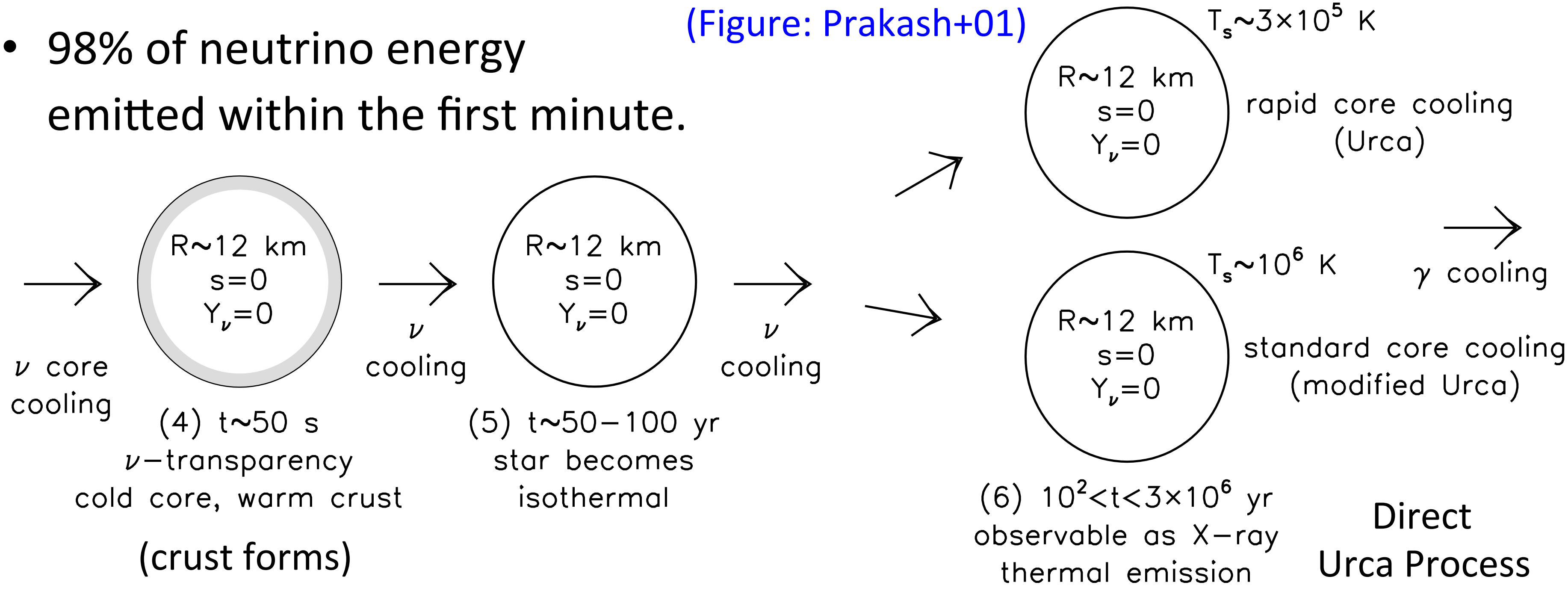


http://www.clccharter.org/maya1/Supernova/Neutron-Artwork.jpg

Evolution from Proto-NS to NS

- 98% of neutrino energy emitted within the first minute.

(Figure: Prakash+01)



(crust forms)

Direct Urca Process

$$n \rightarrow p + e^- + \bar{\nu}_e$$

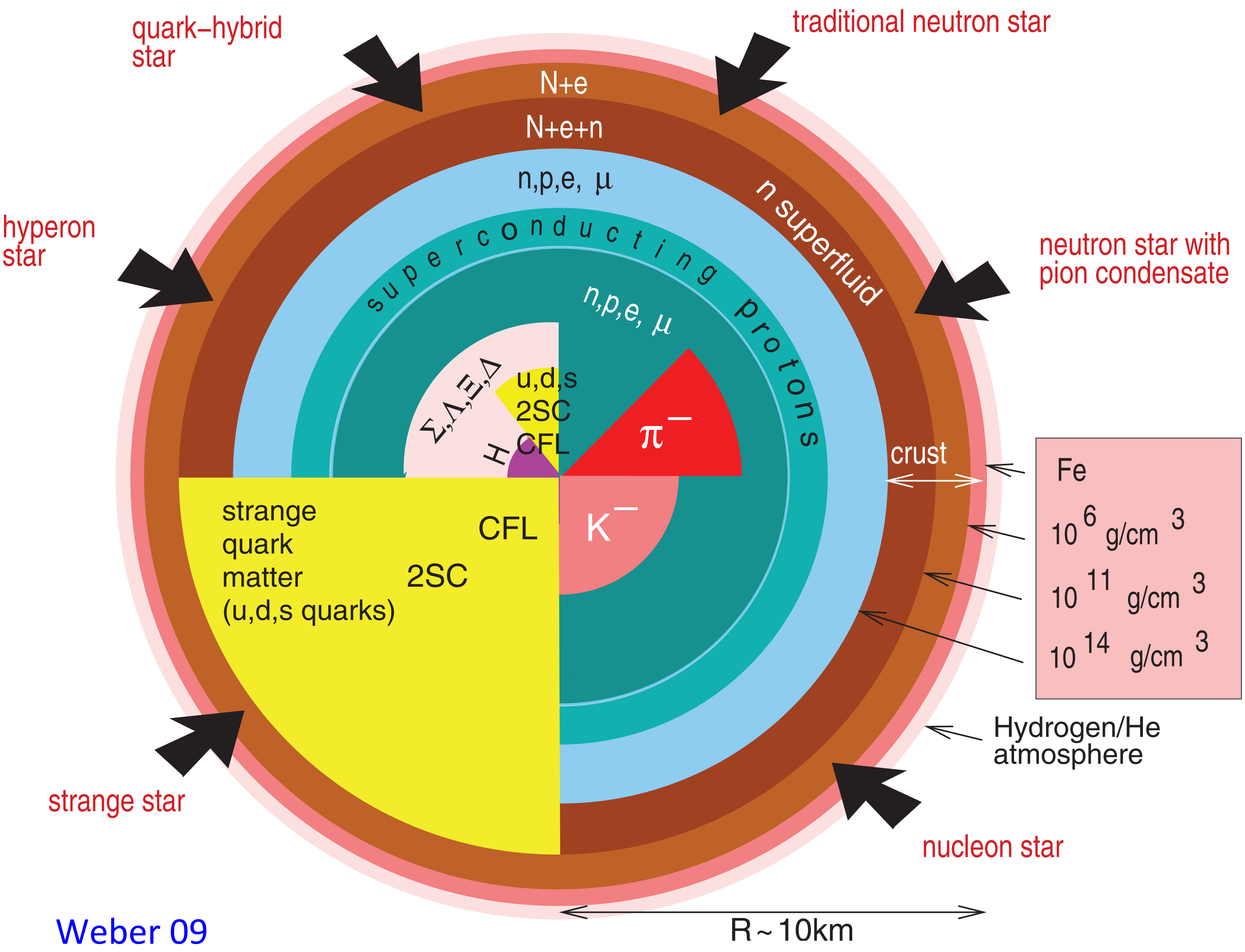
$$p \rightarrow n + e^+ + \nu_e$$

- Cooling dominated by neutrinos for ~10^5 – 10^6 yrs.
- Late time cooling:

  Sensitive to composition and superfluidity.

  (see Page+13 for detailed discussion)

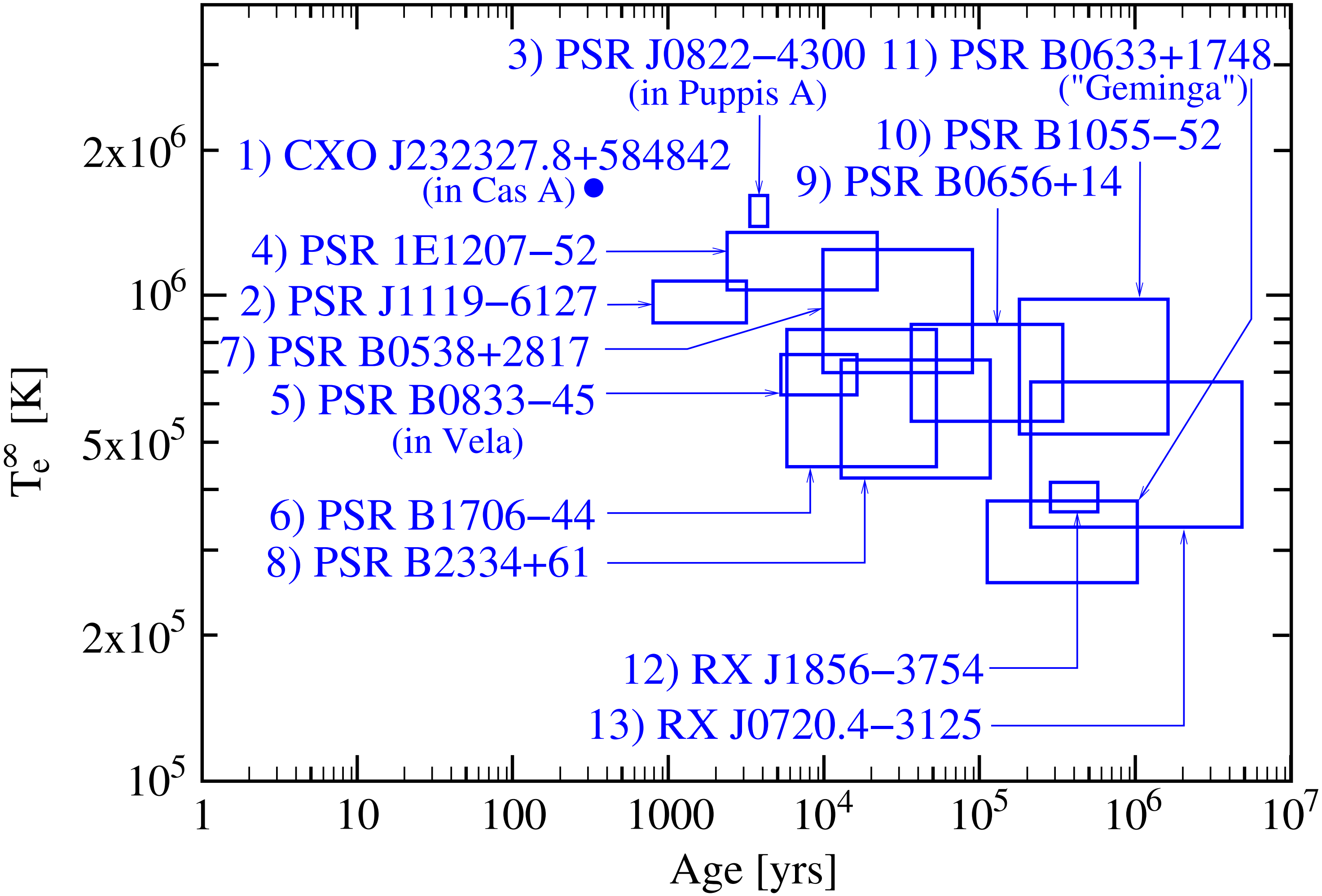
Neutron Star Structure & Composition



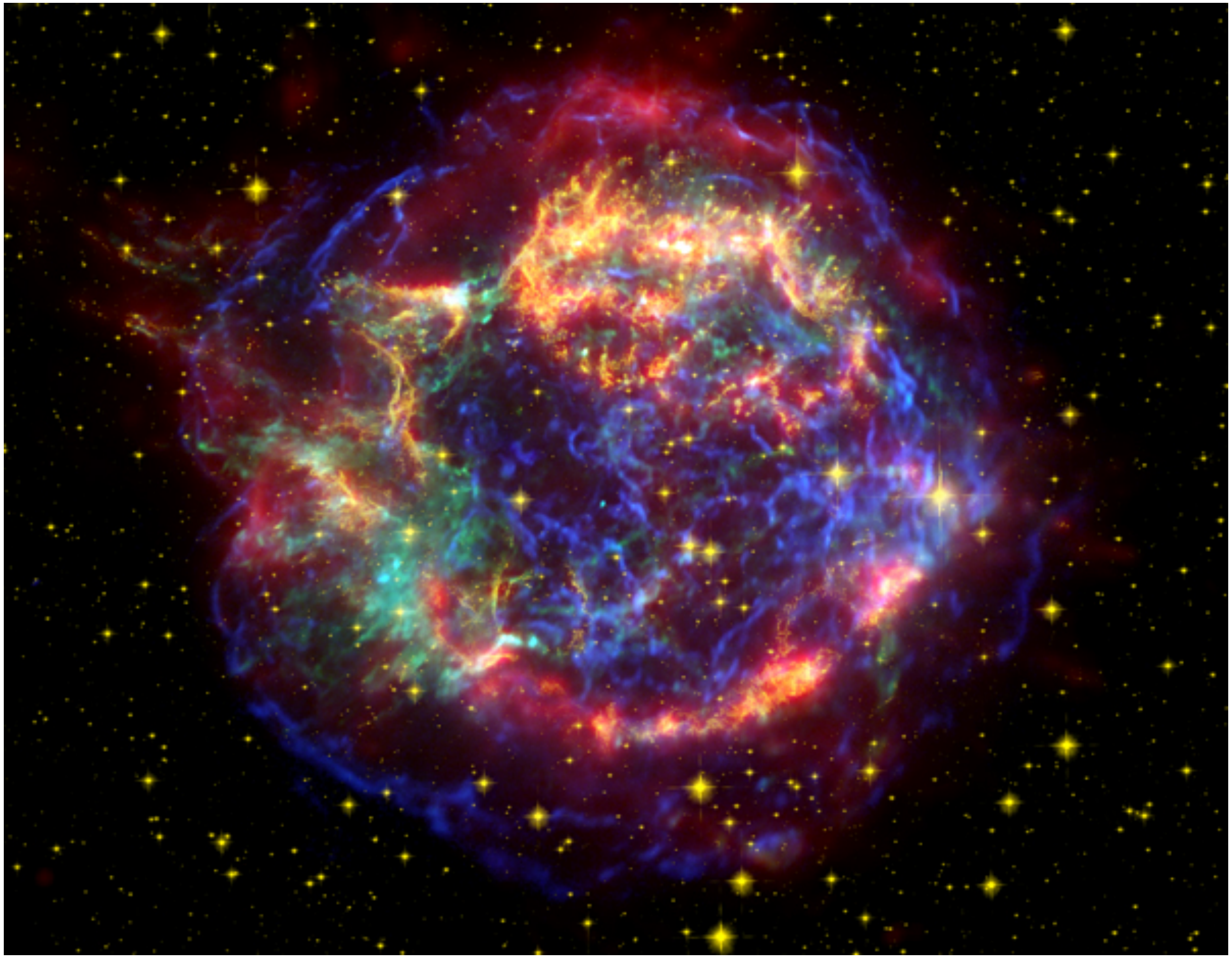
Weber 09

Neutron Star Surface Temperatures

Page+13



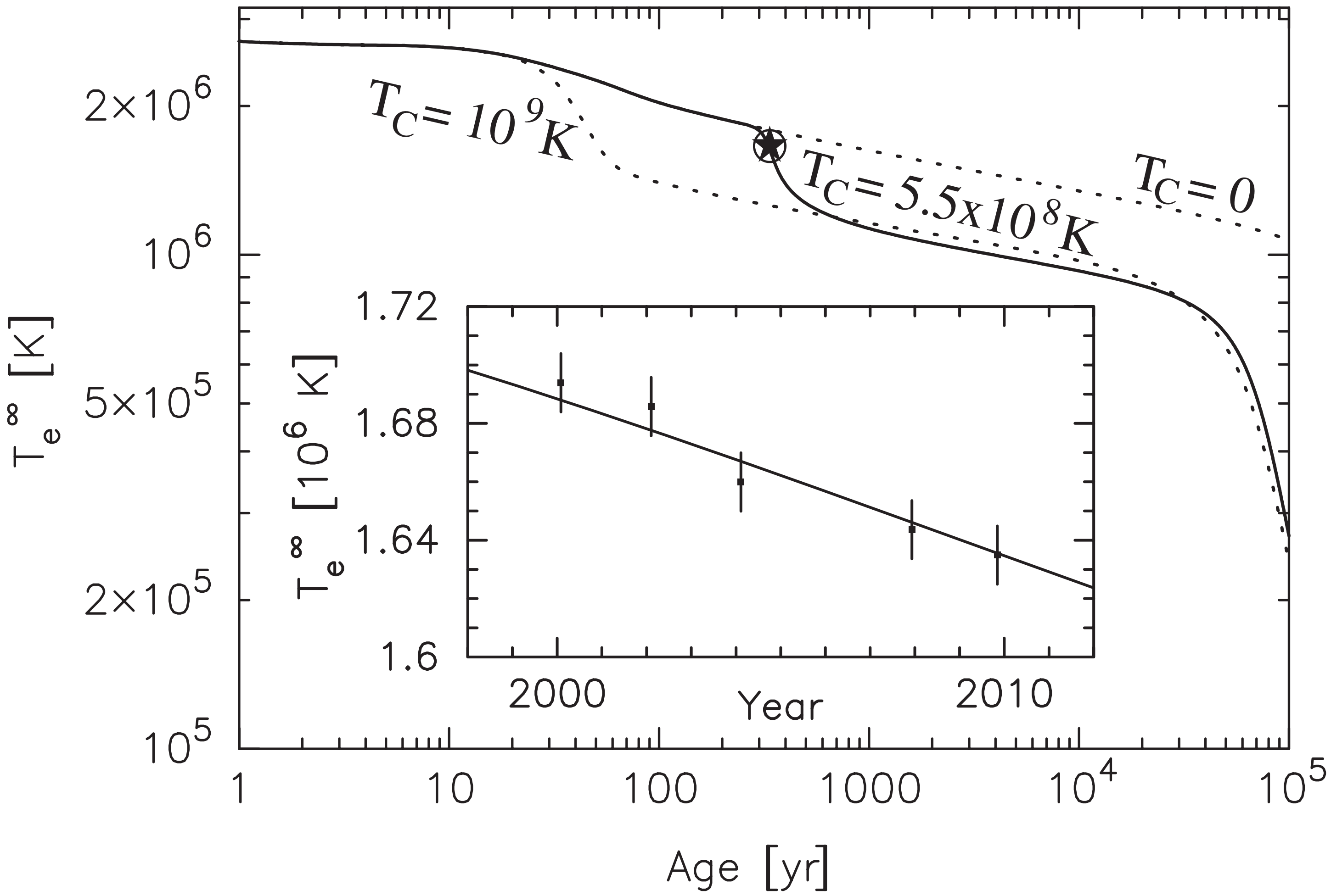
Long-Term Neutron Star Cooling



NS in Cas A SNR: Evidence for rapid cooling –
2.12 x 10^6 K – 2.04 x 10^6 K in 2000-2009 (Heinke & Ho 10)

Long-Term Neutron Star Cooling

Page+11, PRL



Transition to neutron superfluidity:

Increased cooling <- evidence for this in Cas A NS observations!

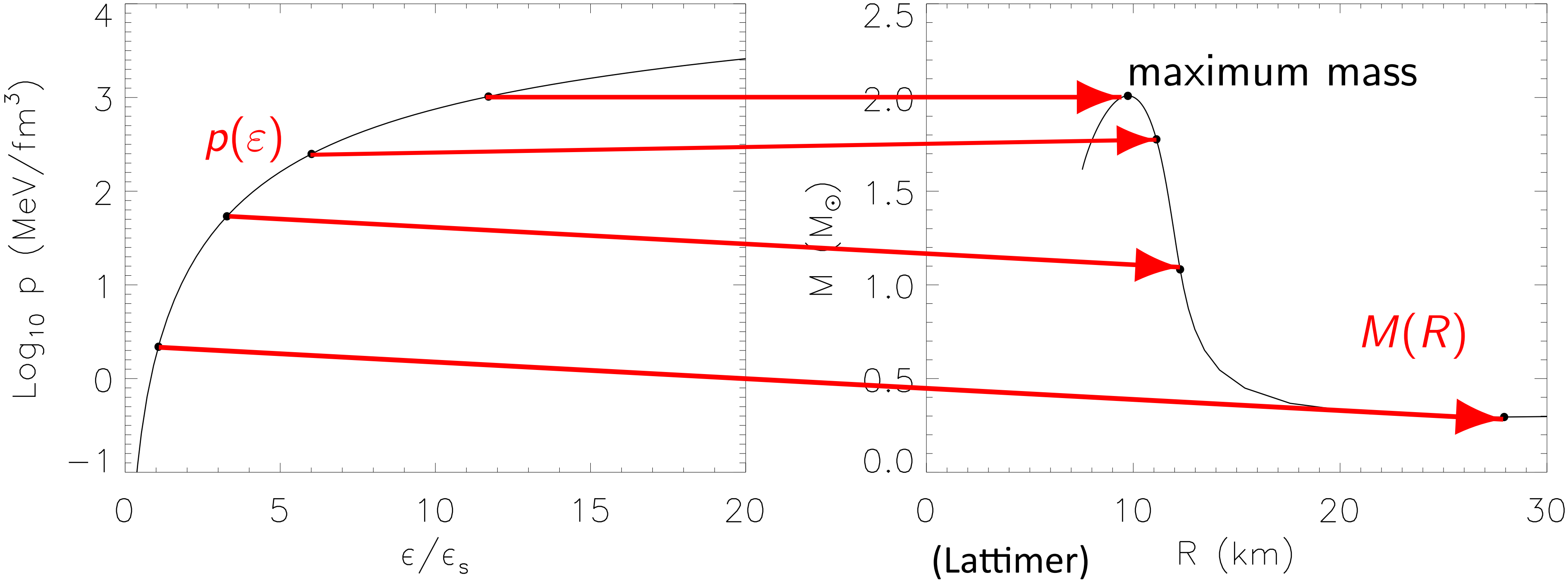
Neutron Star Structure

Tolman-Oppenheimer-Volkoff: GR stellar structure

Energy density

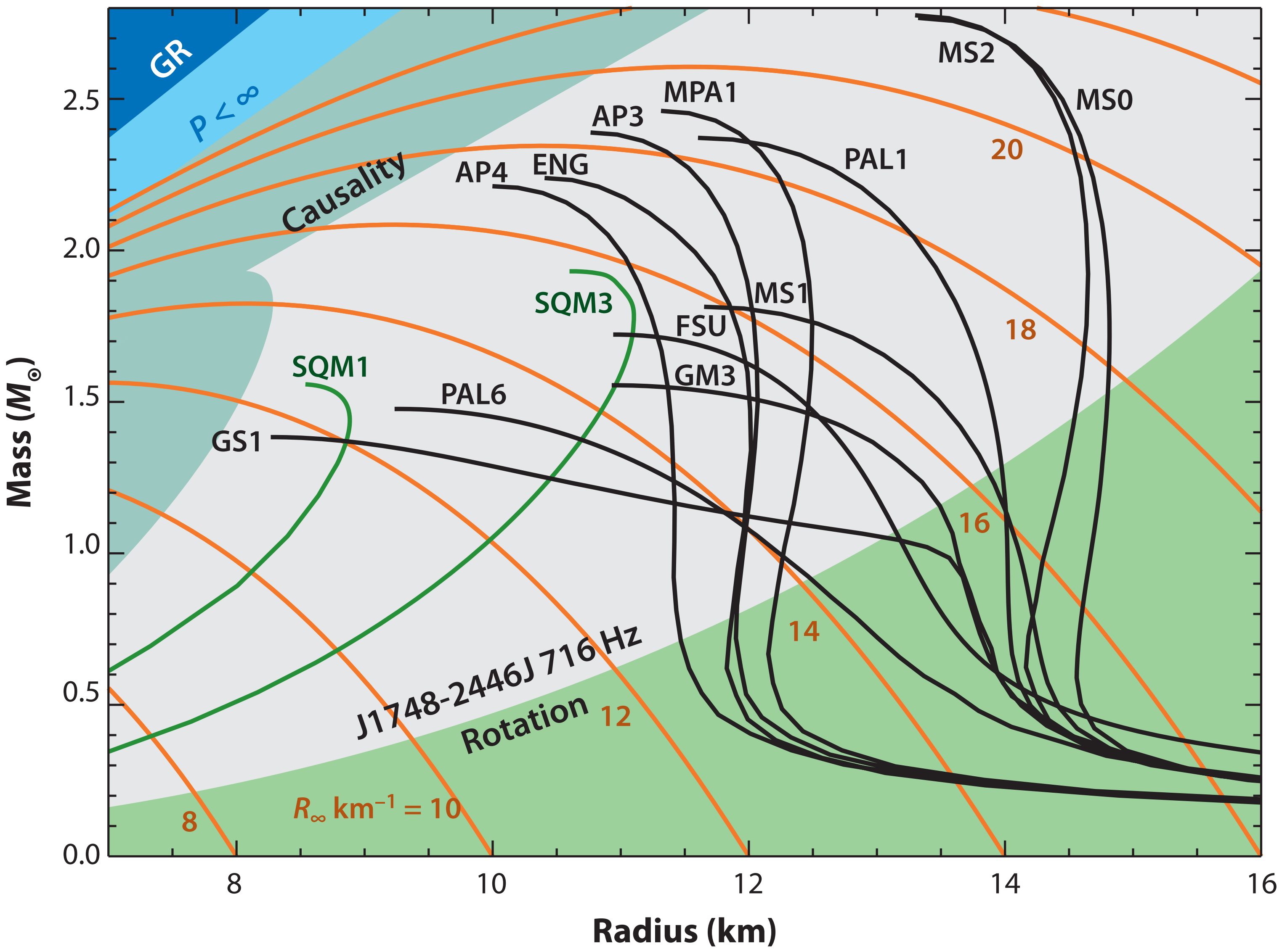
$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m + 4\pi p r^3)(\varepsilon + p)}{r(r - 2Gm/c^2)} \qquad \frac{dm}{dr} = 4\pi\frac{\varepsilon}{c^2}r^2$$

Newtonian limit: $\frac{dP}{dr} = -\frac{GM\rho}{r^2}$



(Lattimer)

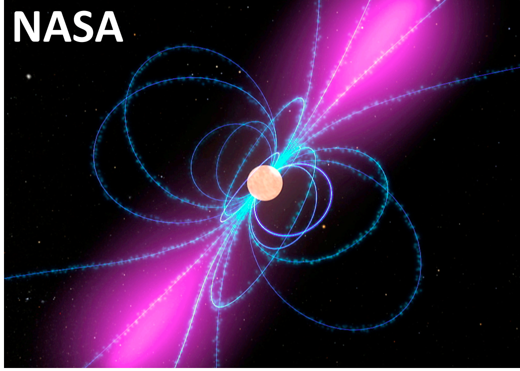
Neutron Star Structure & EOS Constraints



Knowing masses and radii would really help!!!

C. D. Ott @ TAUP Summer School 2013

Neutron Star Masses



Pulsar with binary companion:
Mass function for pulsar

$$f_p = \left(\frac{2\pi}{P}\right)^2 \frac{(a_p \sin i)^3}{G} = \frac{(M_c \sin i)^3}{M^2}$$

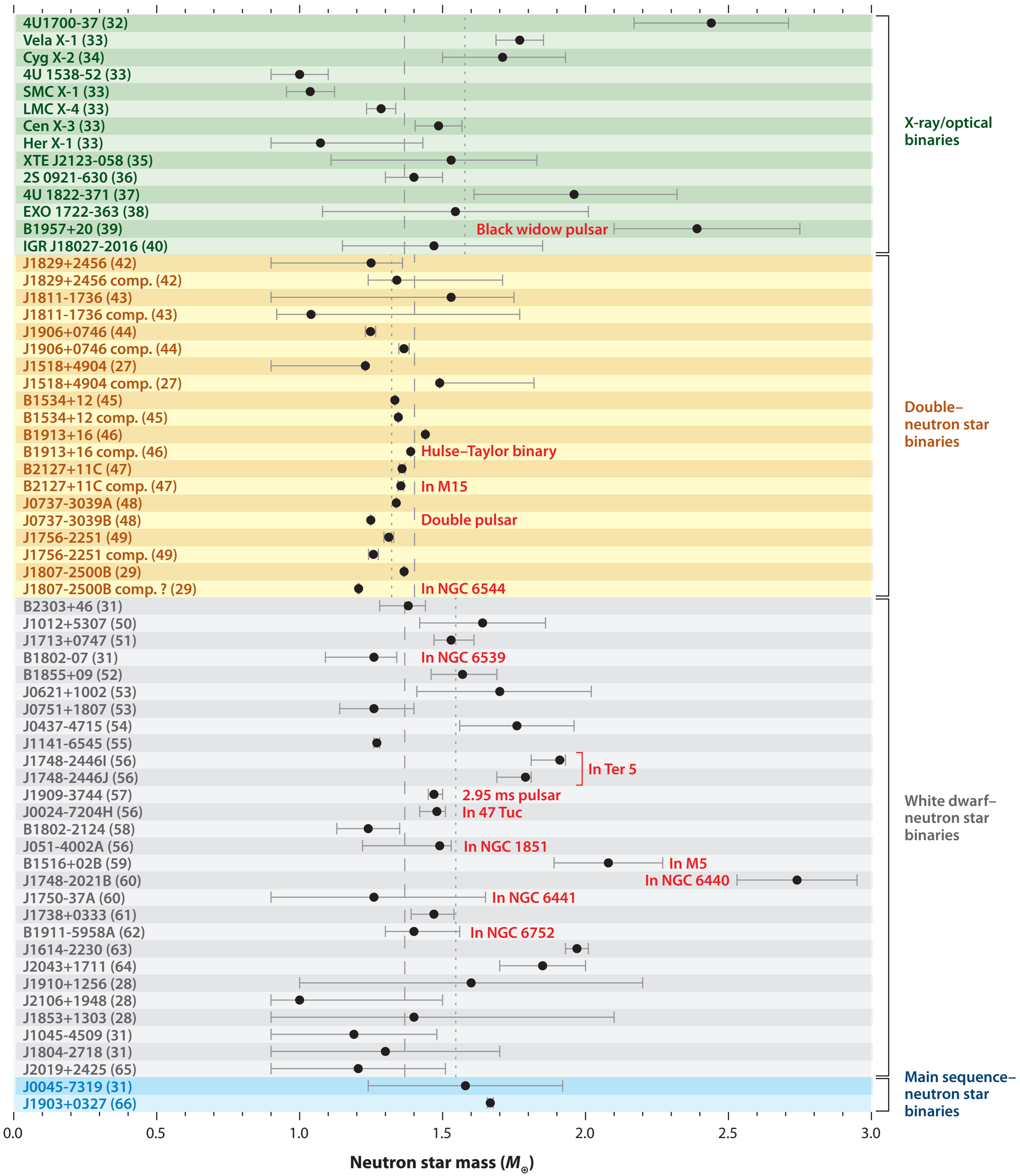
Companion mass

Orbital inclination

Total system mass

- Must know/infer **companion mass** and **inclination** to get M_P.
- Different kinds of binaries:
  X-ray binaries (accreting NSs), double NS binaries, NS–normal-star binaries, NS–WD binaries.
- Companion mass: via stellar models or relativistic effects.
- Inclination: most difficult. In relativistic binaries:
  Shapiro time delay (delay of pulsar pulses by gravity of companion)

Lattimer 12, ARNPS



X-ray binaries

NS+NS

Most massive:

PSR J1614-2230

1.97+-0.04 $M_{\odot}$

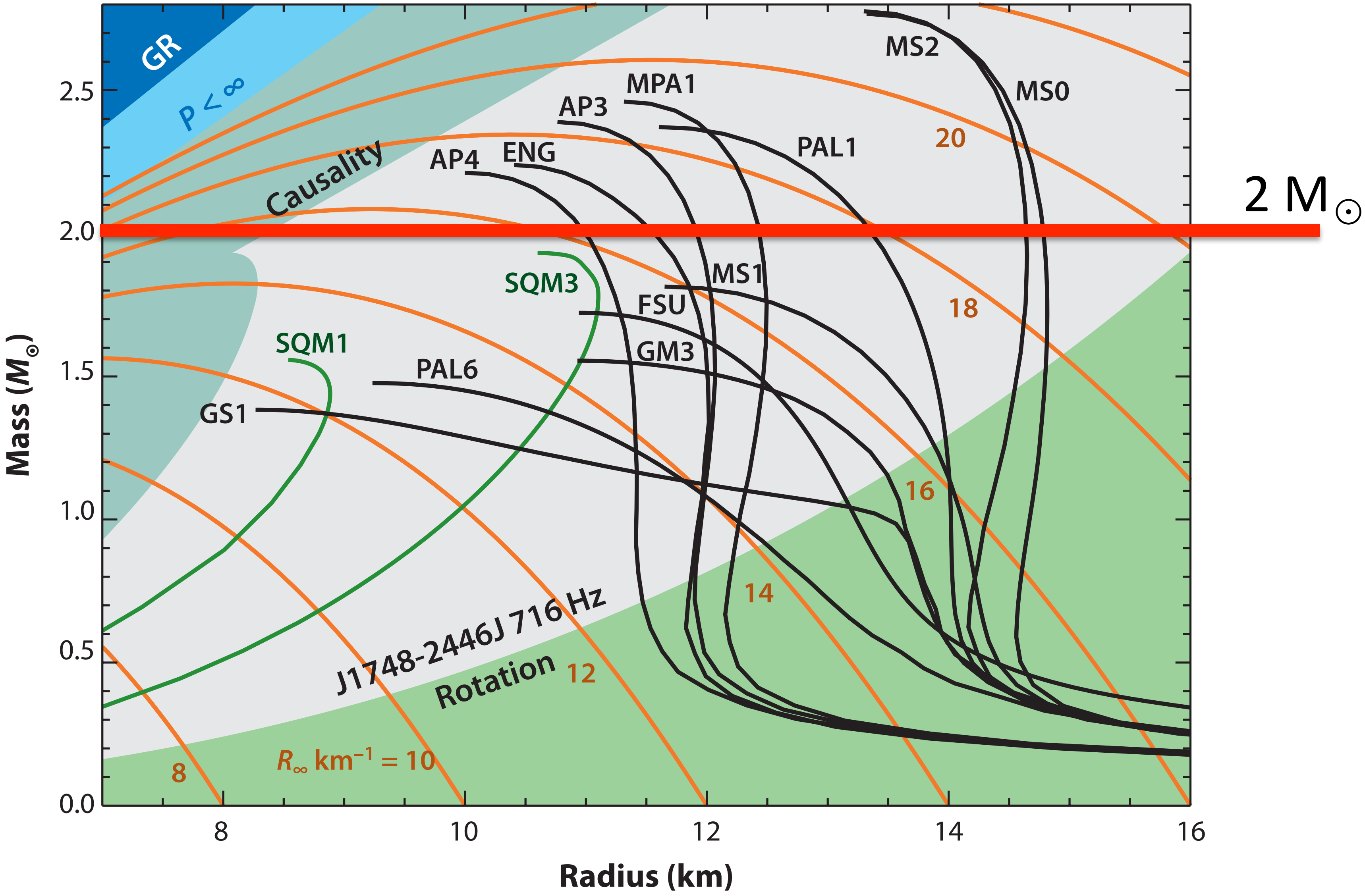
PSR J0348+0432

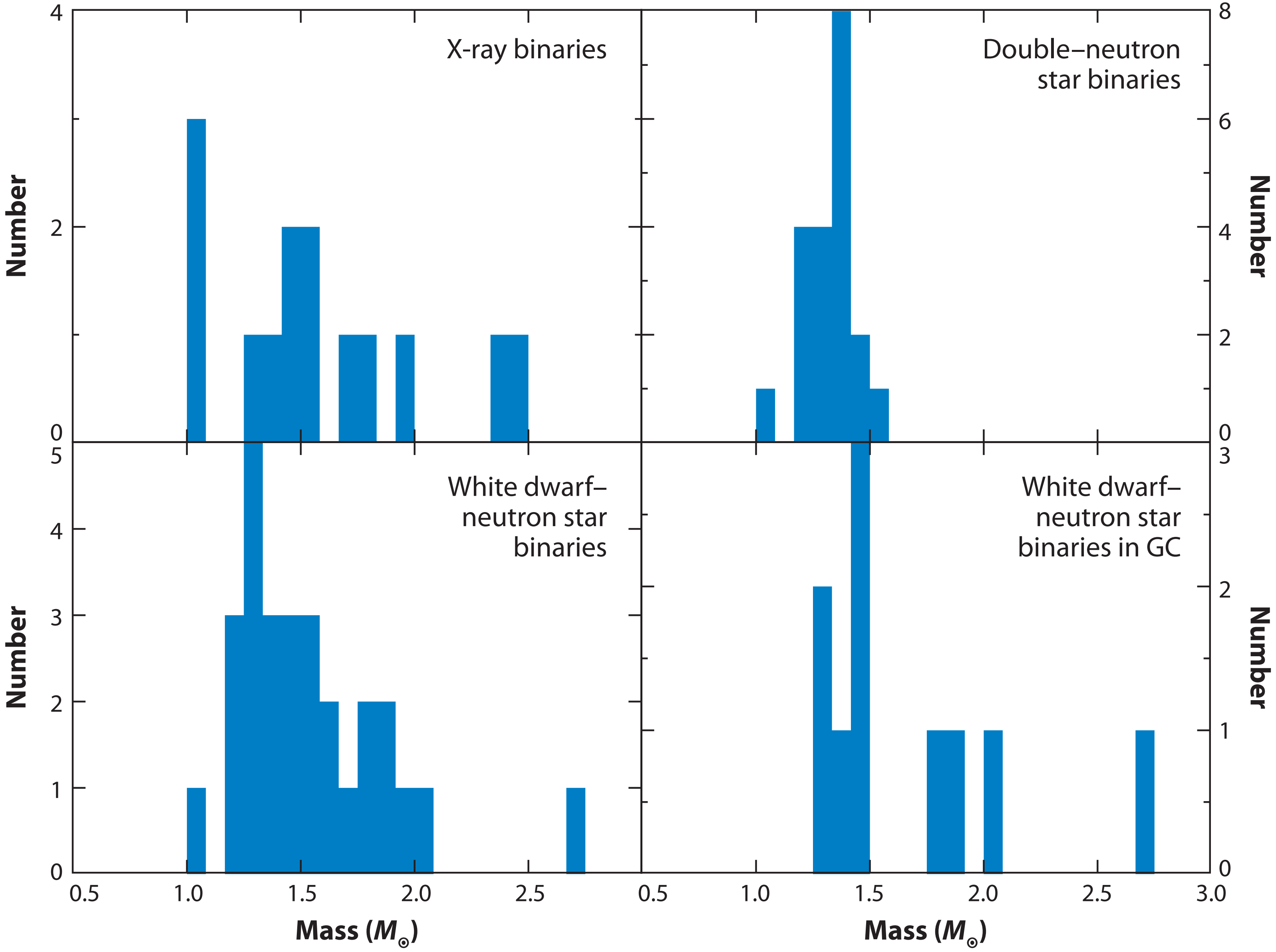
2.01+-0.04 $M_{\odot}$

WD+NS

NS + normal star

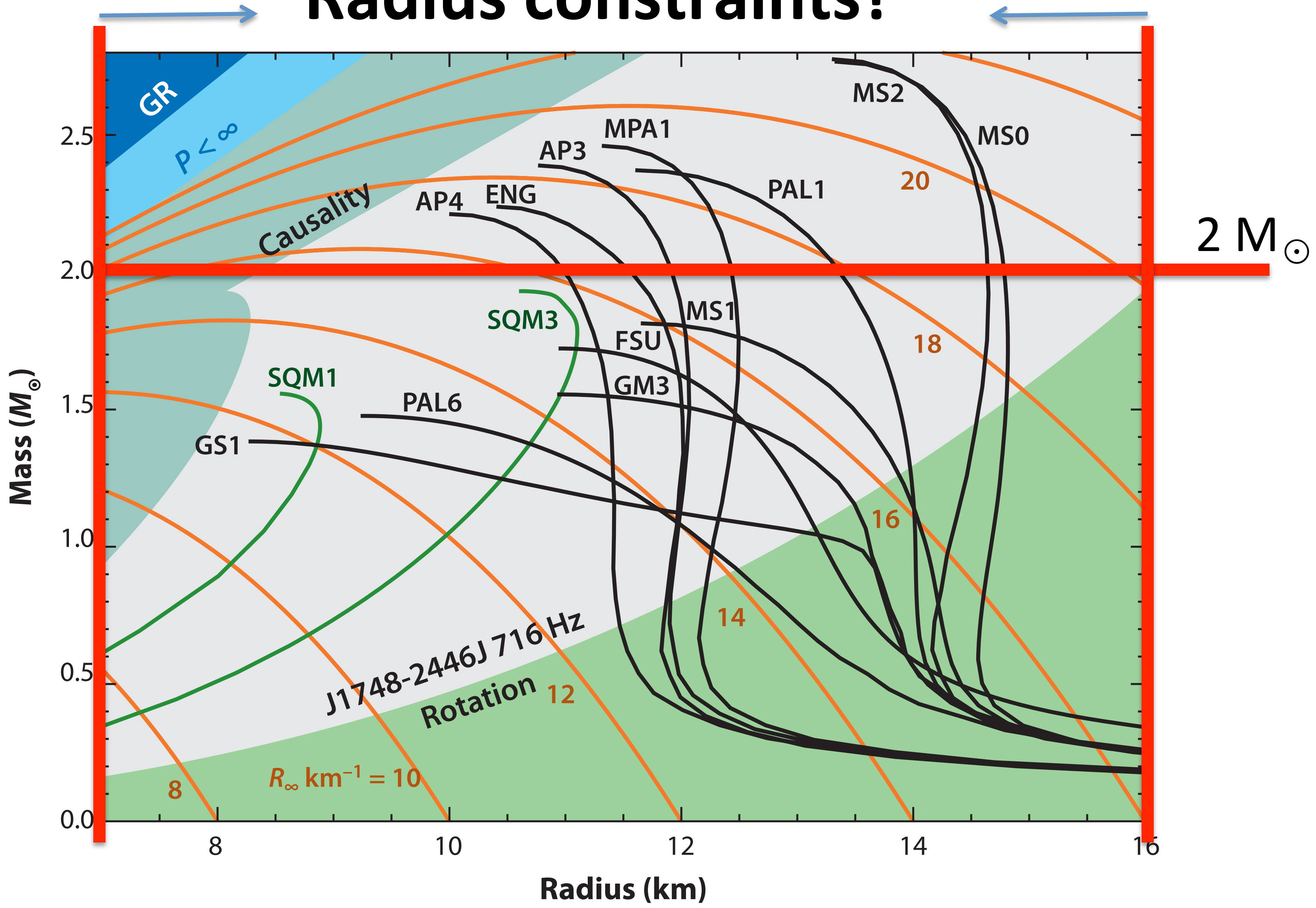
Neutron Star Structure & EOS Constraints





Lattimer 2012

Radius constraints?



Knowing masses and radii would really help!!!

Neutron Star Radii

- So far no robust NS radius (or mass&radius) measurements.
- Approaches: (from Lattimer 12)

1. Thermal X-ray and optical fluxes from isolated and quiescent neutron stars (78).
2. Type I X-ray bursts on neutron star surfaces (79).
3. Quasi-periodic oscillations from accreting neutron stars (80).
4. Spin-orbit coupling, observable through pulsar timing in extremely compact binaries, leading to moments of inertia (81).
5. Pulsar glitches, which constrain properties of neutron star crusts (82).
6. Cooling following accretion episodes in quiescent neutron stars that also constrain crusts (83).
7. Neutron star seismology from X-rays observed from flares from soft γ-ray repeaters (84).
8. Pulse profiles in X-ray pulsars, which constrain M/R ratios due to gravitational light bending (85).

tidal deformation &

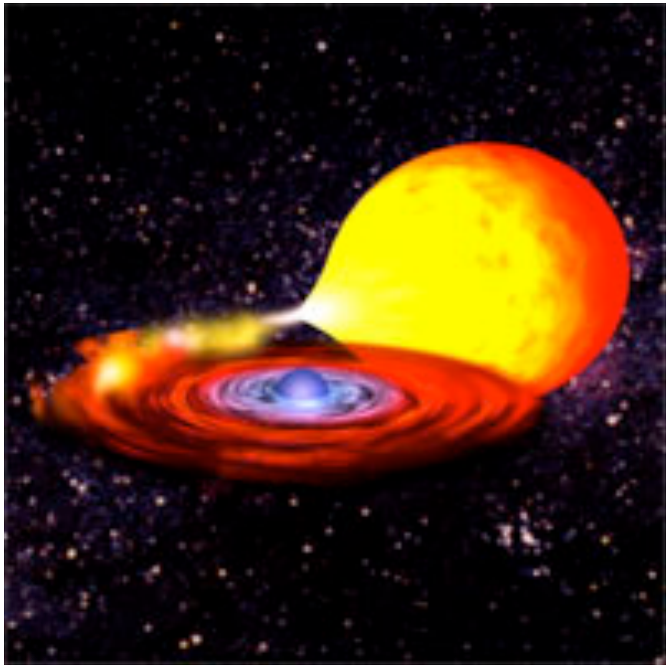
9. Gravitational radiation from tidal disruption of merging neutron stars (7).
10. Neutrino signals from proto–neutron stars formed in Galactic supernovae (72).

Alan Weinstein's talk!

Type I X-Ray Bursts

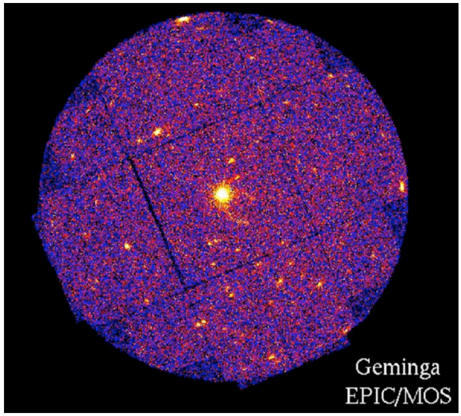
(see Lattimer 12 for review)

NASA



- Unstable He emission on NS surface.
- Rapidly rising X-ray burst (~1s), slow decay (~100s).
- Photosphere expansion:
  Radiation pressure pushes NS atmosphere (=photosphere), balances gravity.
- Observation + atmosphere models + distance
  -> radius and mass (but model dependent)

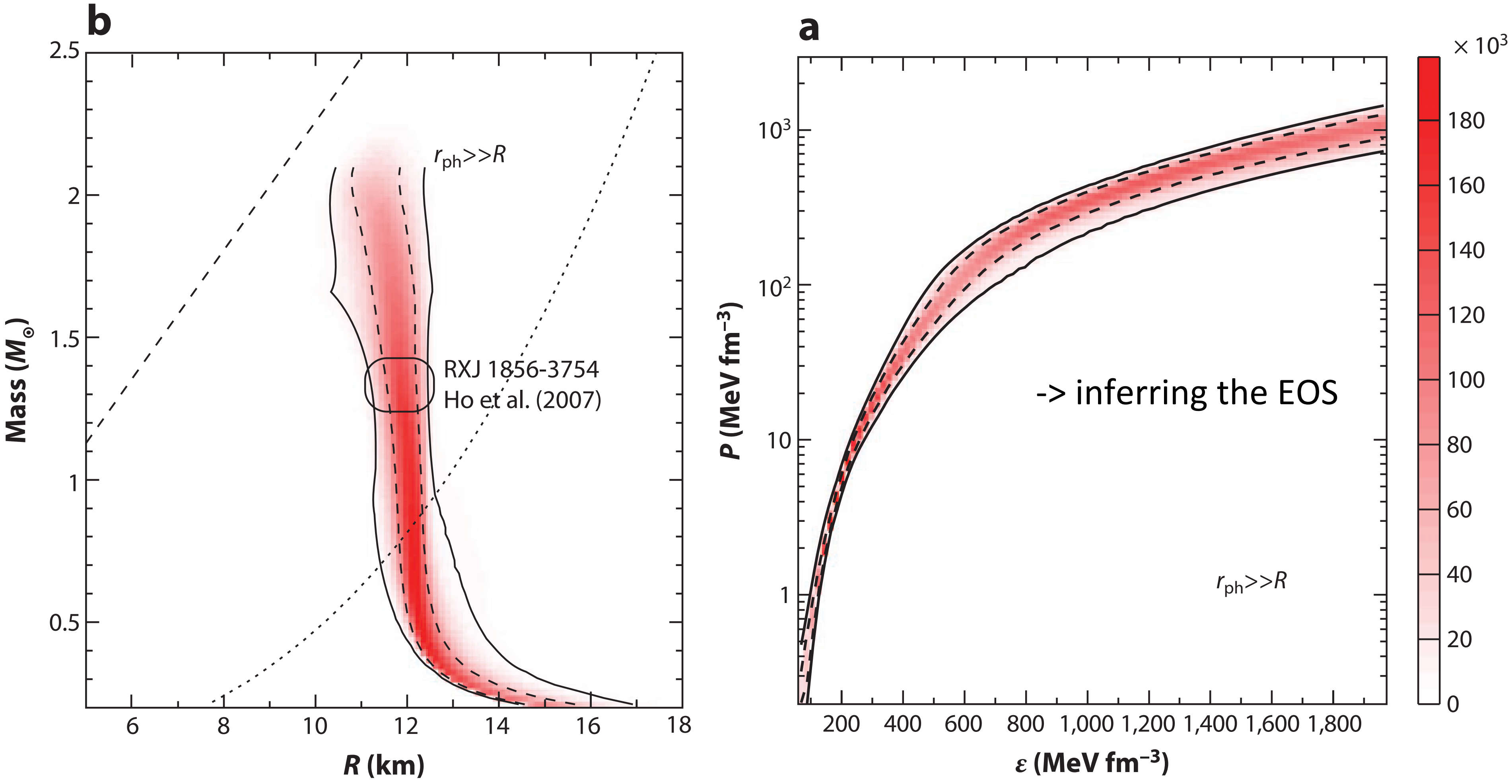
Quiescent NSs



XMM/Newton

- (Almost) Black-body UV/X-ray emission of young neutron stars.
- Depends on NS atmosphere composition, magnetic field, galactic UV/X-ray absorption. Need to know distance.
- Fits based on atmosphere models give radius and mass estimates.

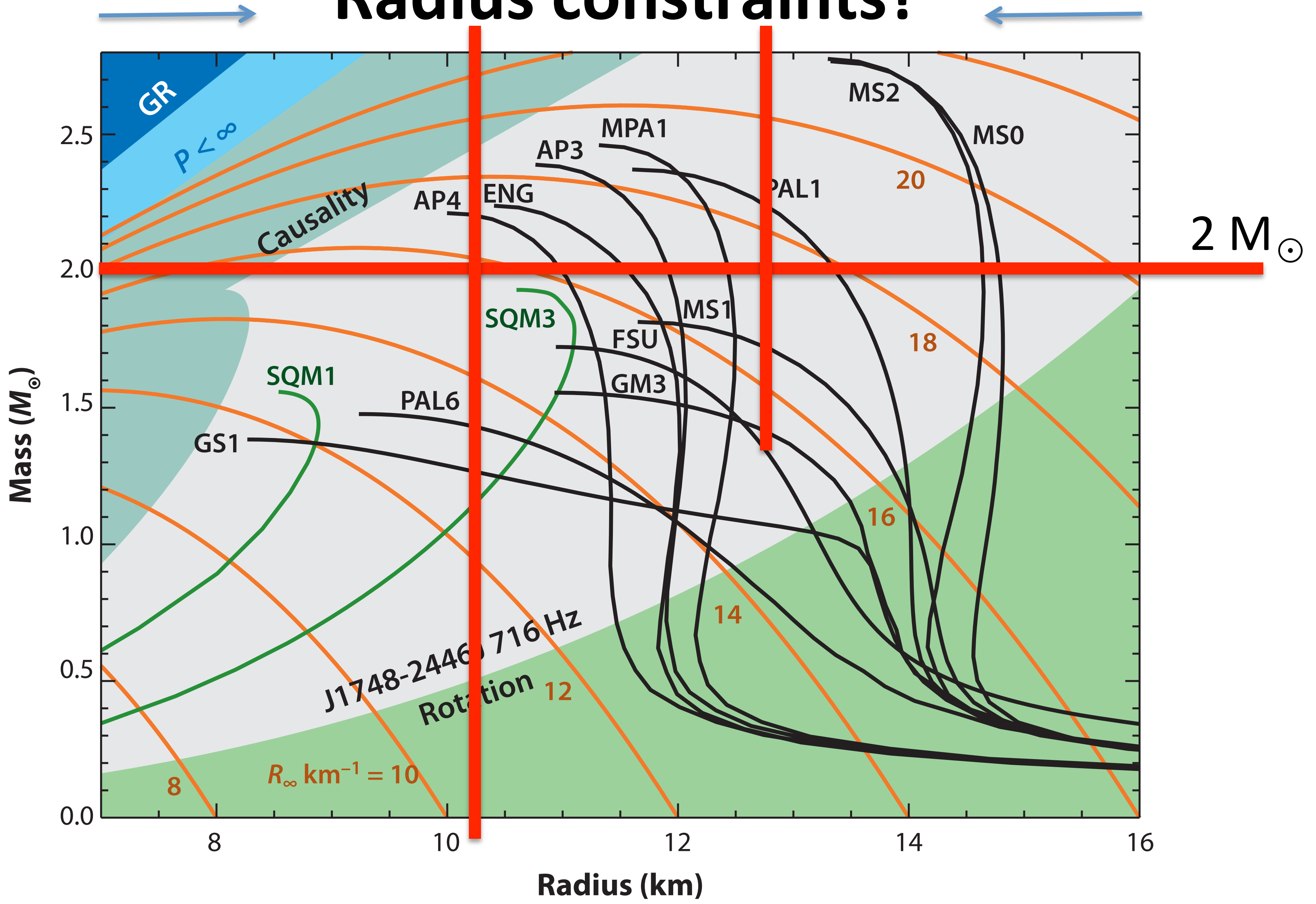
Neutron Star Masses & Radii



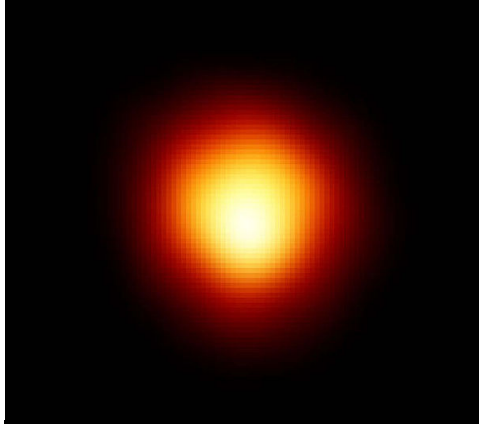
Statistical Analysis of observational data: Steiner+10,+12, Lattimer 12

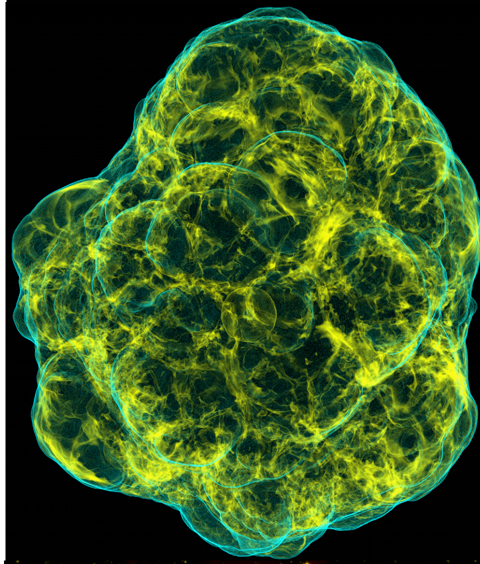
Warning: Does not fix model dependence of M, R estimates!

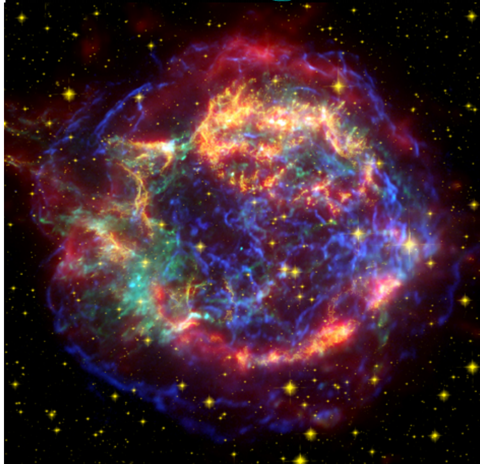
Radius constraints?



Summary & Conclusions of Lecture II





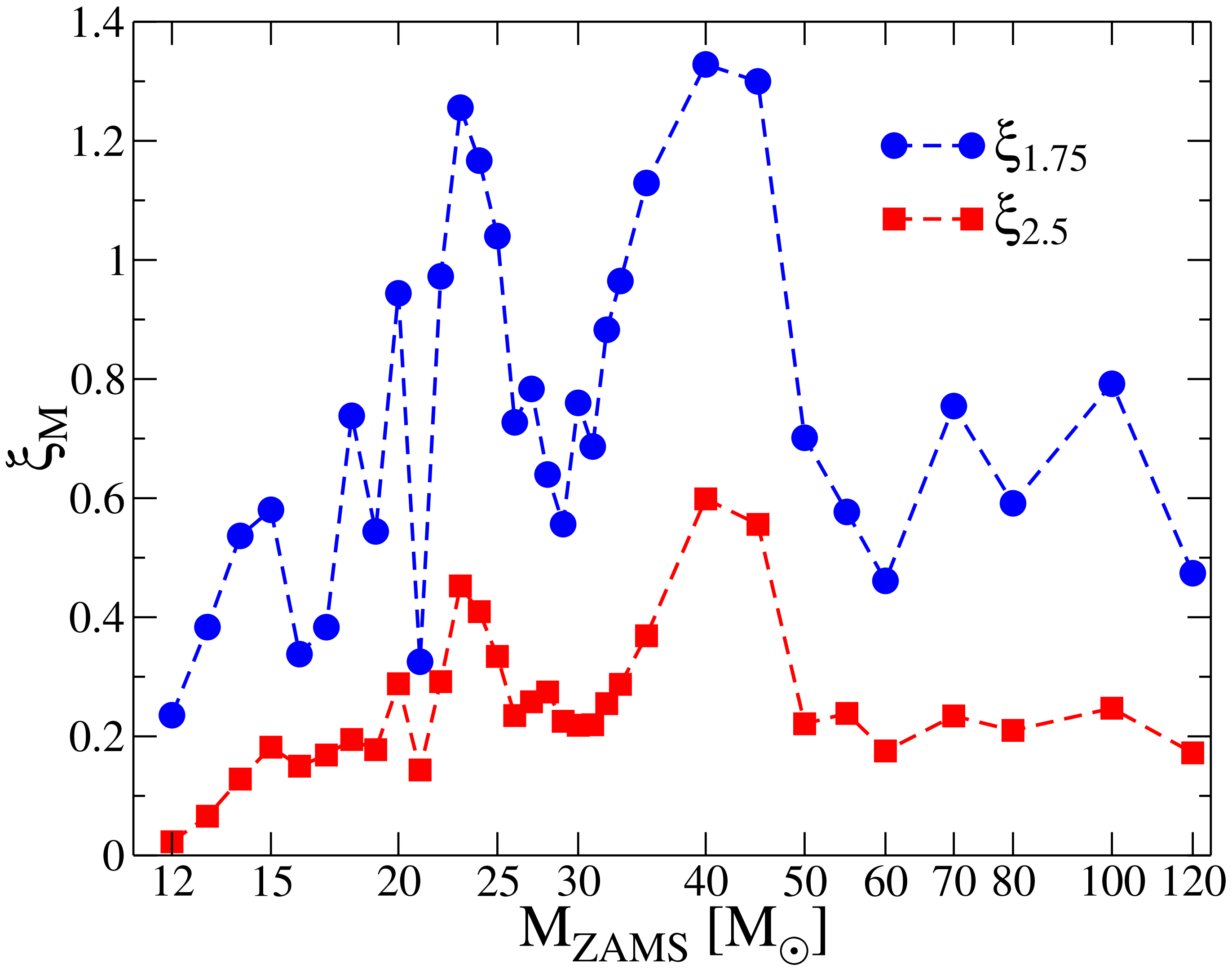


- Basics of core-collapse supernova theory on solid foundation; details to be worked out.
- Multi-dimensional neutrino mechanism best bet for blowing up ordinary massive stars. **Next:** complete 3D models.
- Increasingly better constraints on the nuclear EOS via NS mass and radius constraints. Also: laboratory constraints & better theory.
- **The next galactic core-collapse supernova has already exploded.** (But its GWs/neutrinos/EM waves better not get here until 2015+.)
- Neutrinos and GWs probe supernova dynamics and thermodynamics -> nuclear/neutrino physics.

Bonus Slides

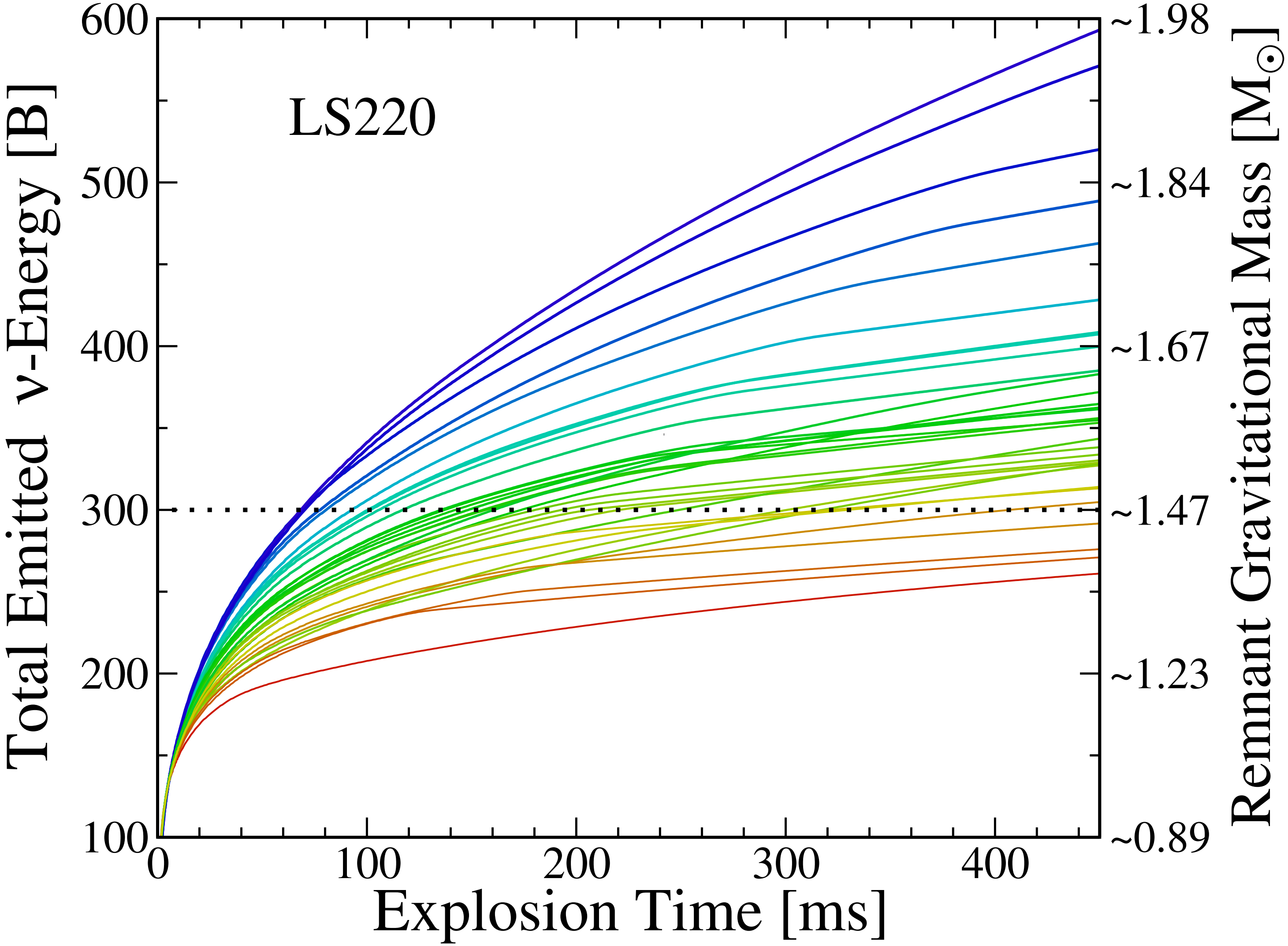
Compactness Parameter & Stellar Mass

O'Connor & Ott 2013

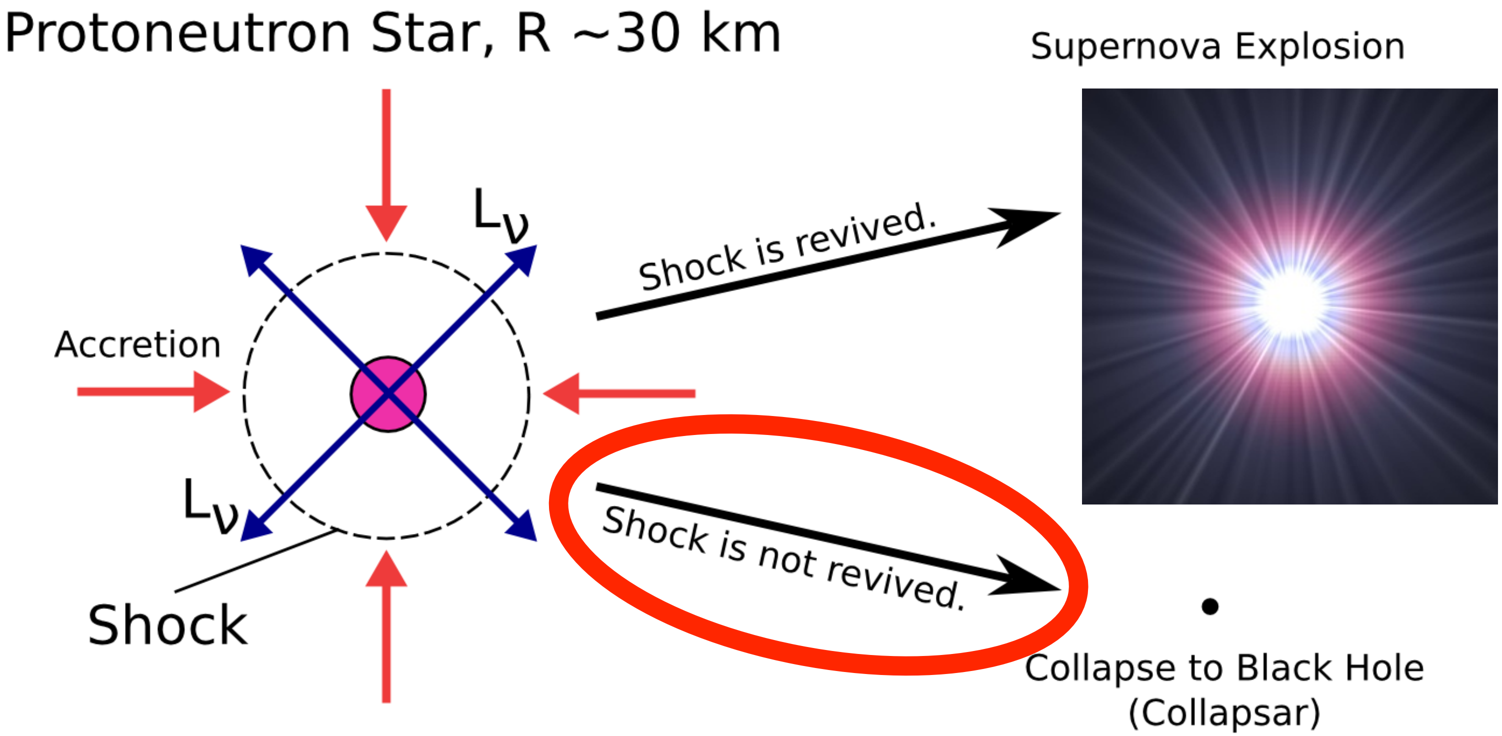


Remnant Mass from Neutrinos

O'Connor & Ott 2013

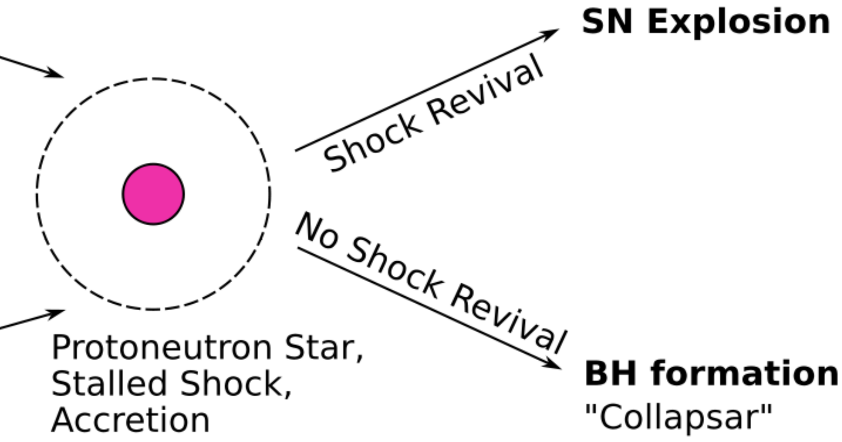


When things go wrong...



Black Hole Formation

It’s not actually quite that simple...



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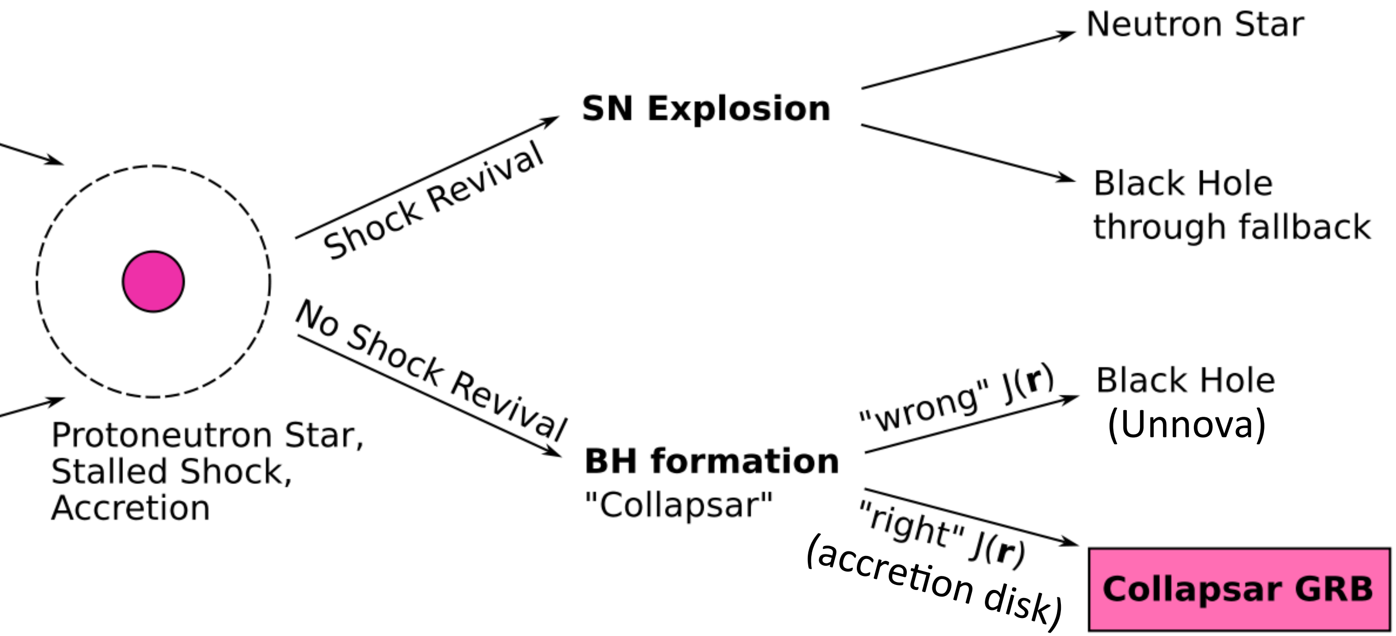


Protoneutron Star,
Stalled Shock,
Accretion

Shock Revival

No Shock Revival

SN Explosion

Neutron Star

Black Hole
through fallback

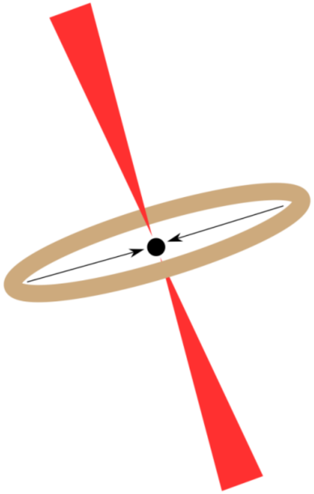
BH formation
"Collapsar"

"wrong" J(**r**)

Black Hole
(Unnova)

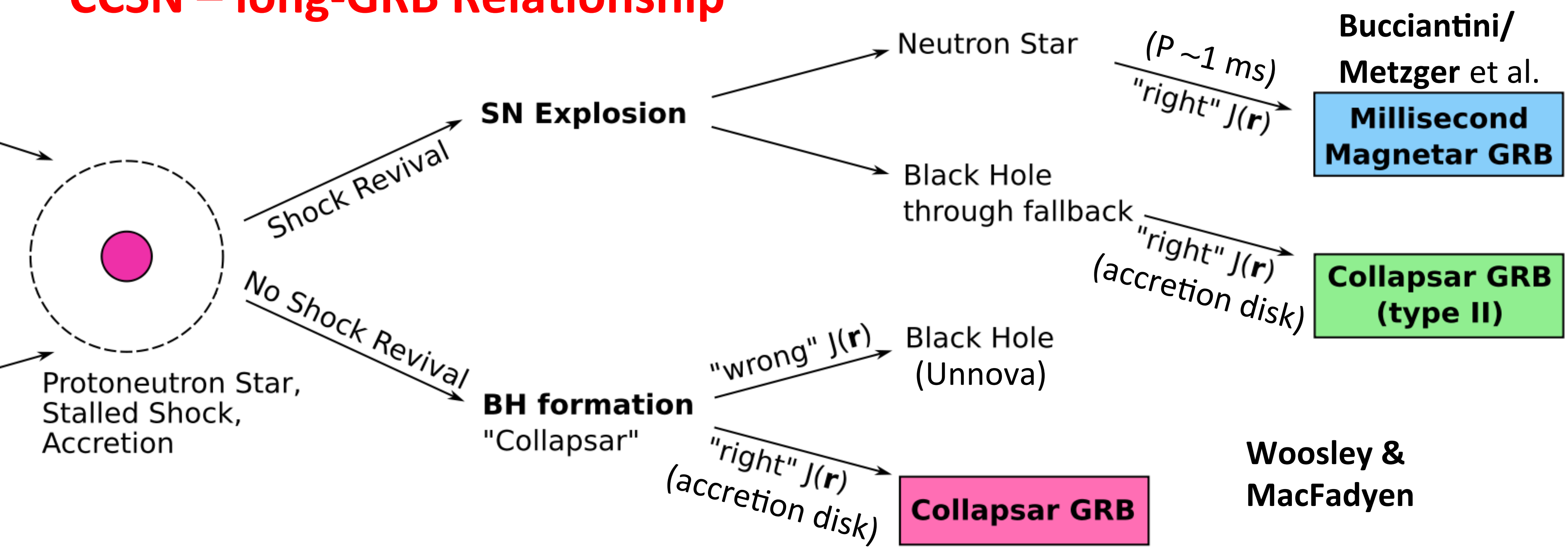
"right" J(**r**)
(accretion disk)

Collapsar GRB



It’s not actually quite that simple...

CCSN – long-GRB Relationship



Protoneutron Star,
Stalled Shock,
Accretion

Shock Revival

SN Explosion

Neutron Star

(P ~1 ms)
"right" J(**r**)

Bucciantini/
Metzger et al.

Millisecond Magnetar GRB

Black Hole
through fallback

"right" J(**r**)
(accretion disk)

Collapsar GRB (type II)

No Shock Revival

BH formation
"Collapsar"

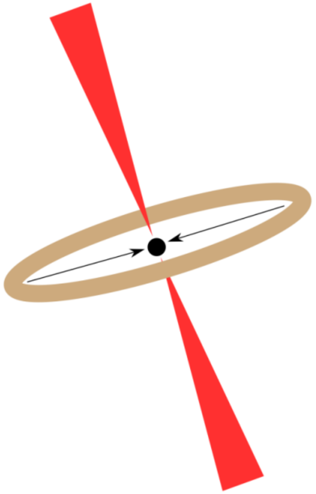
"wrong" J(**r**)

Black Hole
(Unnova)

"right" J(**r**)
(accretion disk)

Collapsar GRB

Woosley &
MacFadyen



- What is the long GRB central engine?
  What decides which branch is taken?
  -> currently not understood!

A few more words on making BHs:

$$X = \frac{1}{\sqrt{1 - \frac{2GM}{r^2}}}$$

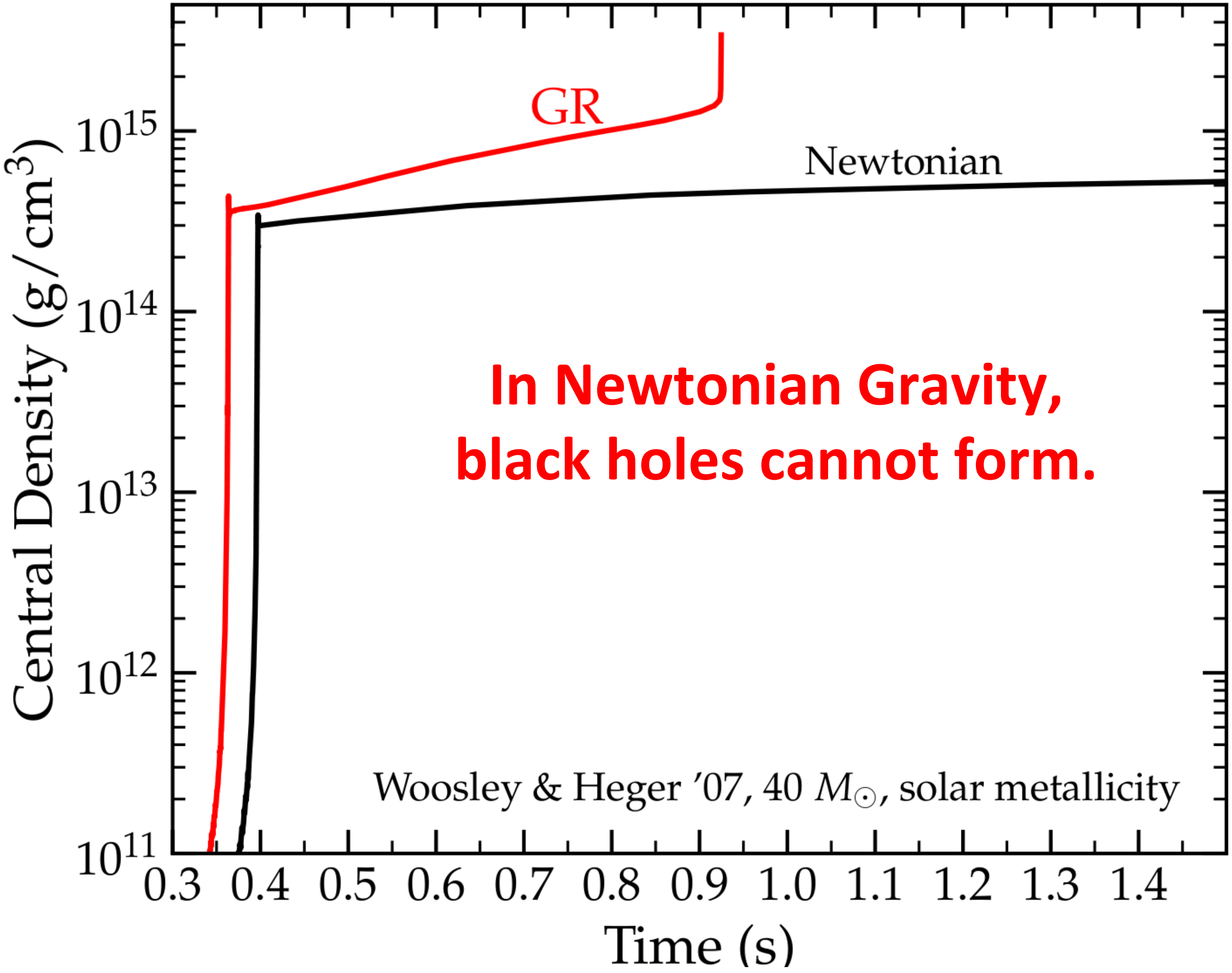
- First things first:
  The is NO such thing as direct (“prompt”) collapse to a black hole in ordinary massive stars (i.e. ZAMS mass 10 – 130 M_{Sun})

- Black hole formation may happen in 3 ways:
  - No explosion; proto-NS accretes more M than can be supported by EOS. Maximum mass: controlled by EOS, temperature + rotation.
  - Successful explosion, but much fallback accretion.
  - Successful explosion, but hadron/quark phase transition during cooling.

GR Hydrodynamics

$$G^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$

General Relativity: Why bother?



In Newtonian Gravity, black holes cannot form.

Maximum Neutron Star Mass: Dependence on the Nuclear EOS

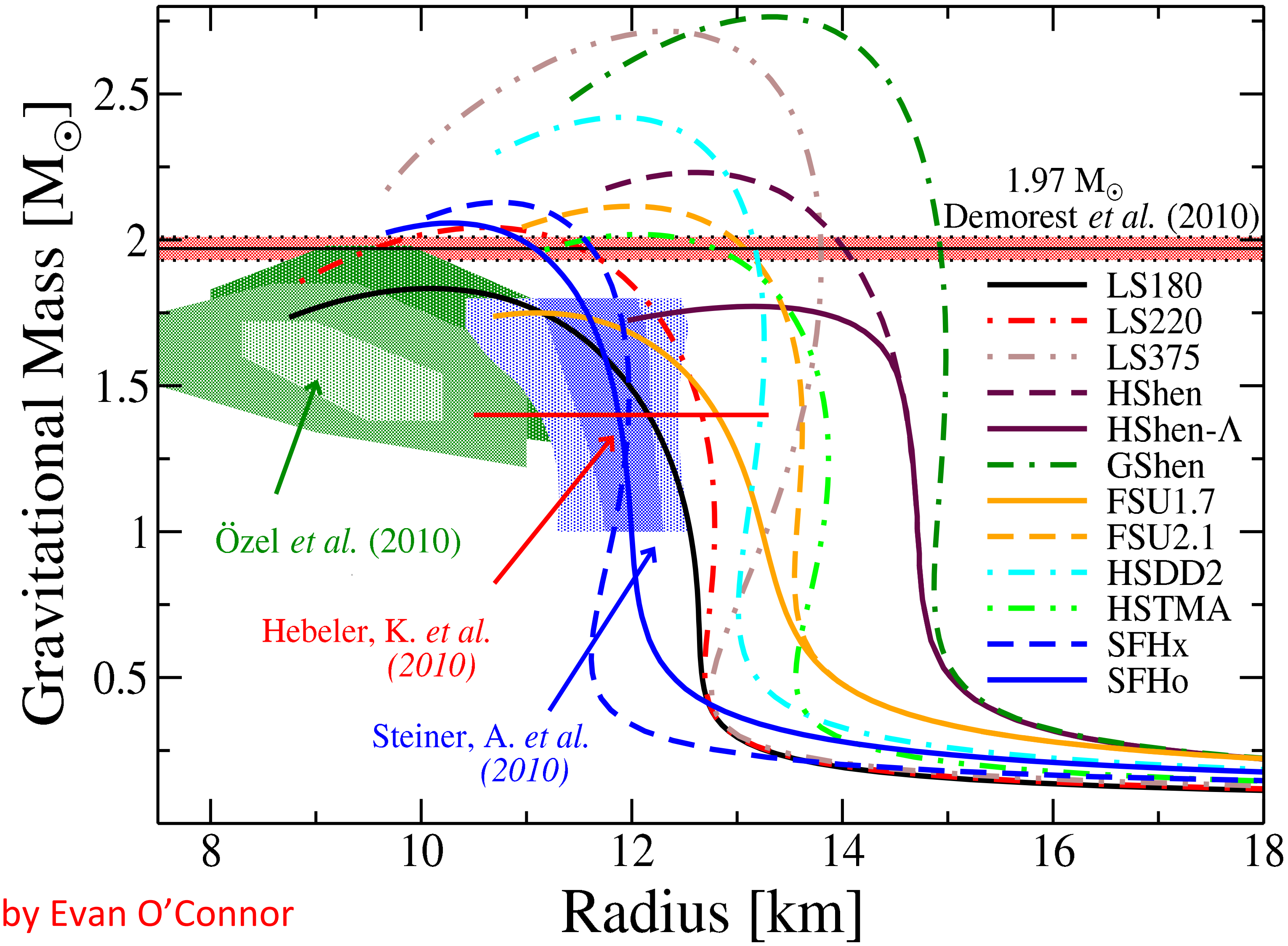
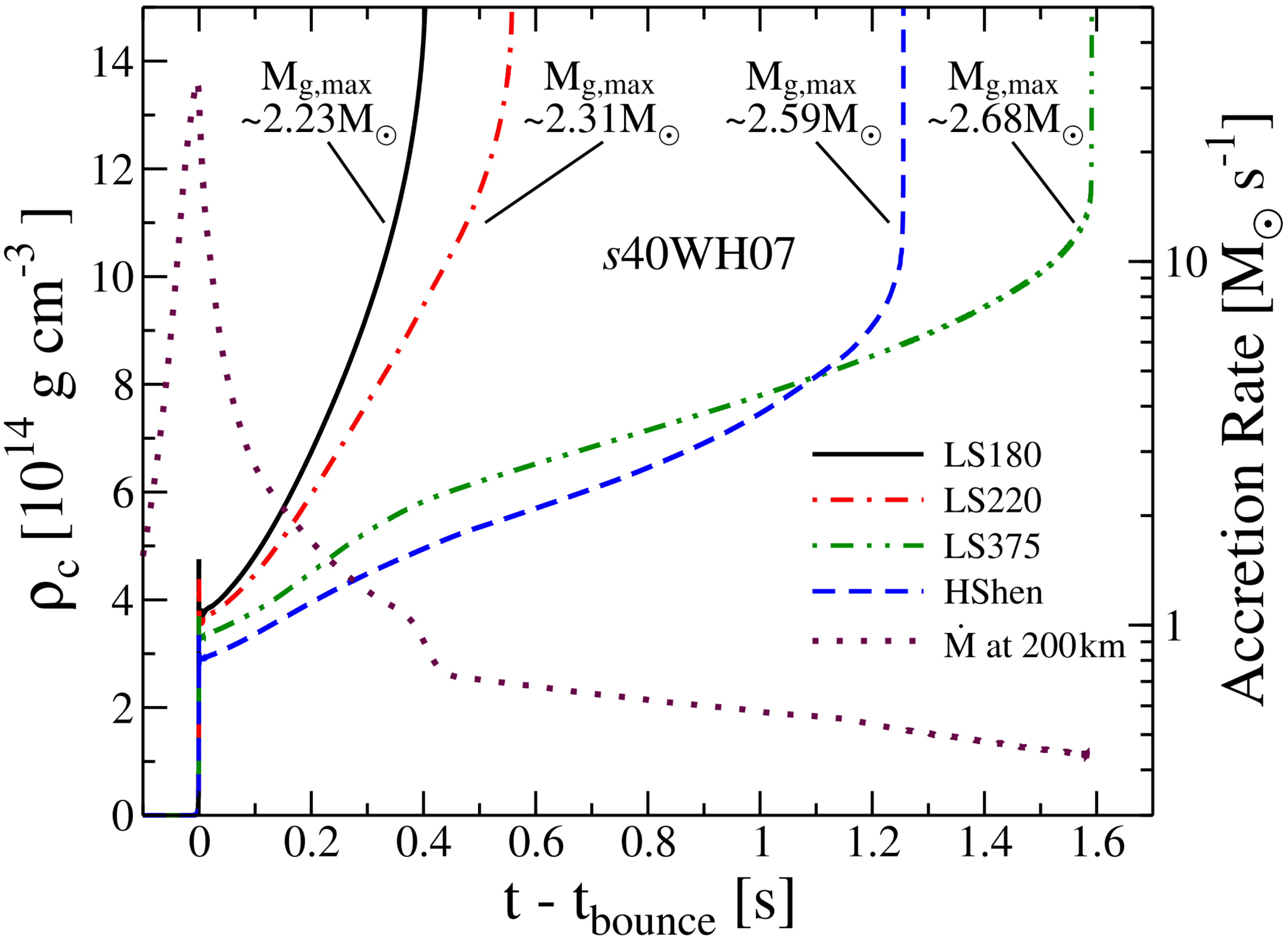


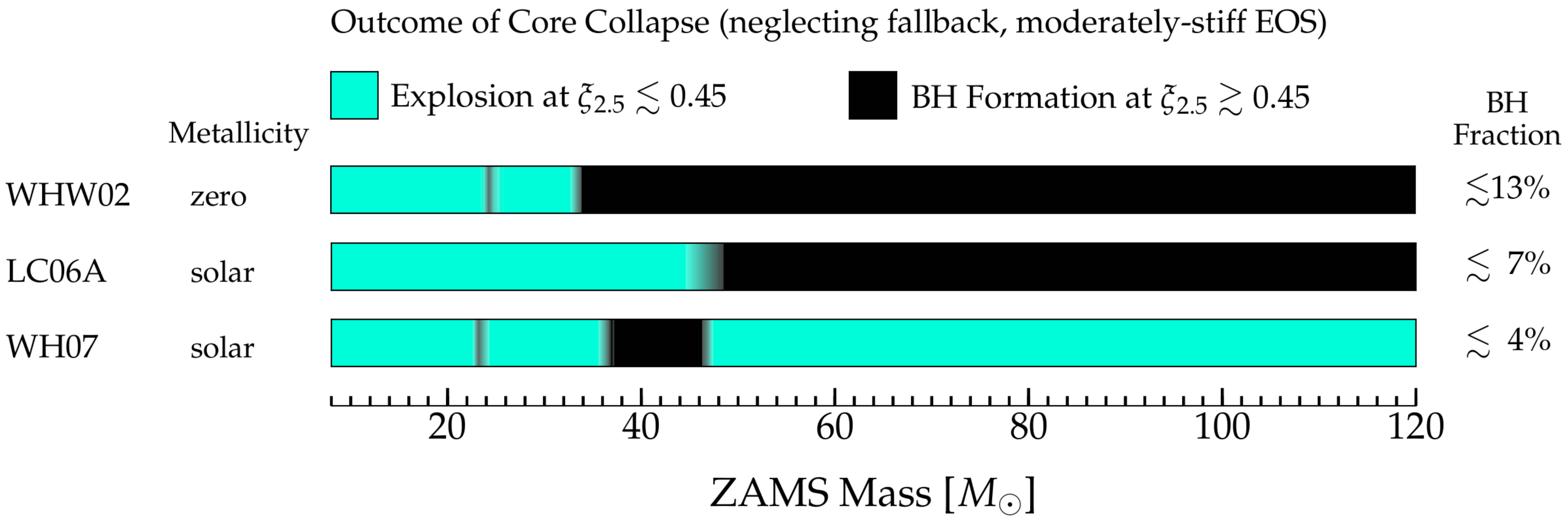
Figure by Evan O'Connor

Dependence on the Nuclear Equation of State



What Stars make Black Holes?

(O'Connor & Ott 2011; see also Ugliano et al. 2012)



Large uncertainty at solar metallicity: Physics of mass loss highly uncertain!

Electron Capture

Simplest case: Capture on free protons, neutrinos escape

$$e^- + p \overset{(W)}{\longrightarrow} \nu_e + n \qquad \mu_{\nu_e} = 0$$

capture if $\mu_e > \mu_n - \mu_p$

At zero T, non-degenerate nucleons: $\mu_e > 939.565\,\mathrm{MeV} - 938.272\,\mathrm{MeV} = 1.293\,\mathrm{MeV}$

In core collapse: Capture typically at μ_e ~ >10 MeV -> excess energy given to ν.

Capture rates: (see, e.g., Bethe et al. 1979, Bethe 1990, Burrows, Reddy & Thompson 2006)

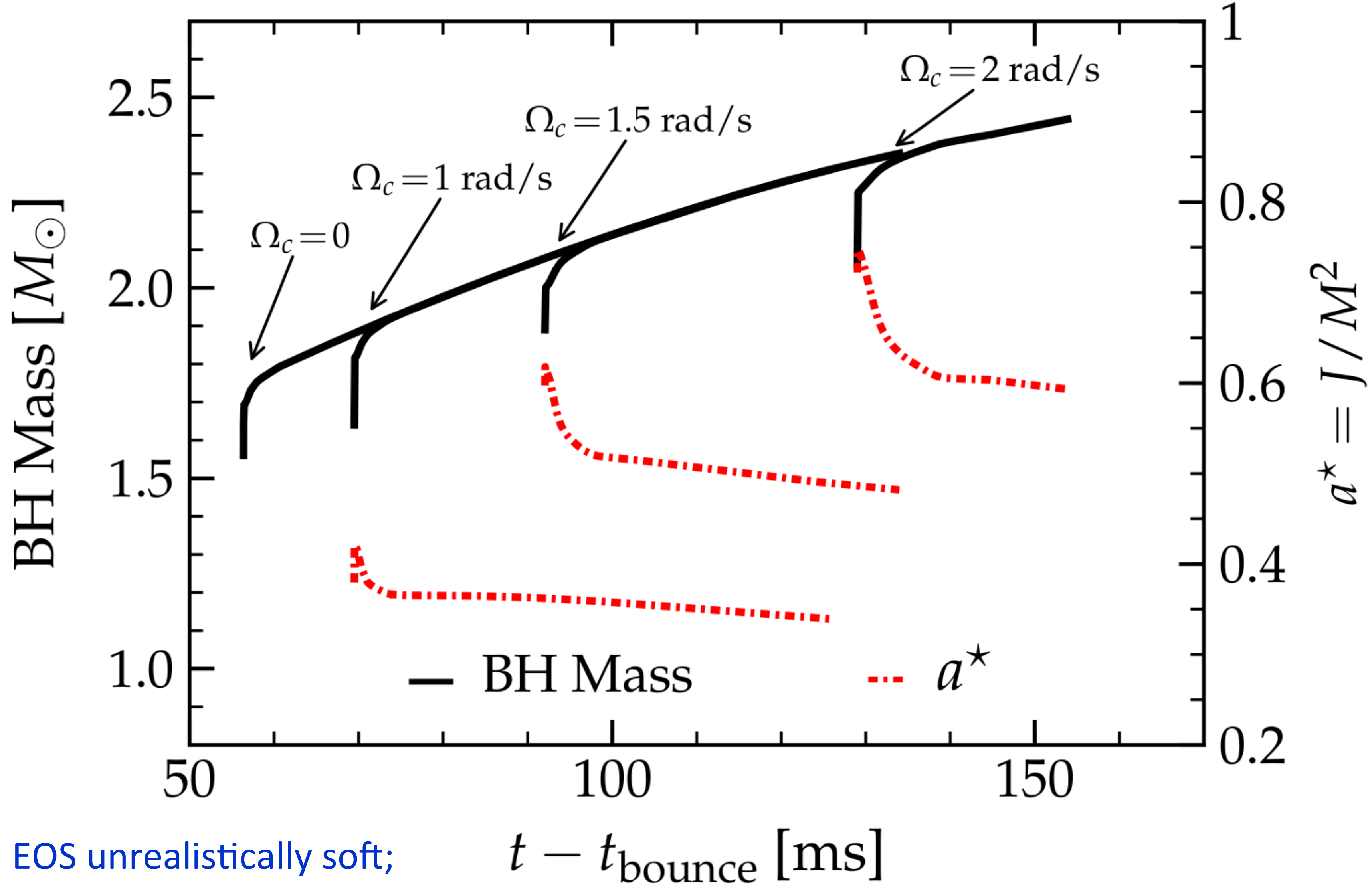
$$\frac{\partial}{\partial t} Y_e \propto \mu_e^5 \propto \rho^{5/3}$$

Complications:

- Capture on nuclei more complicated; can be blocked due to neutron shells filling up.
- Pauli blocking of low-energy states, since neutrinos don't exactly leave immediately.

Nascent BH Spin and Mass Evolution

Ott+ 2011, PRL

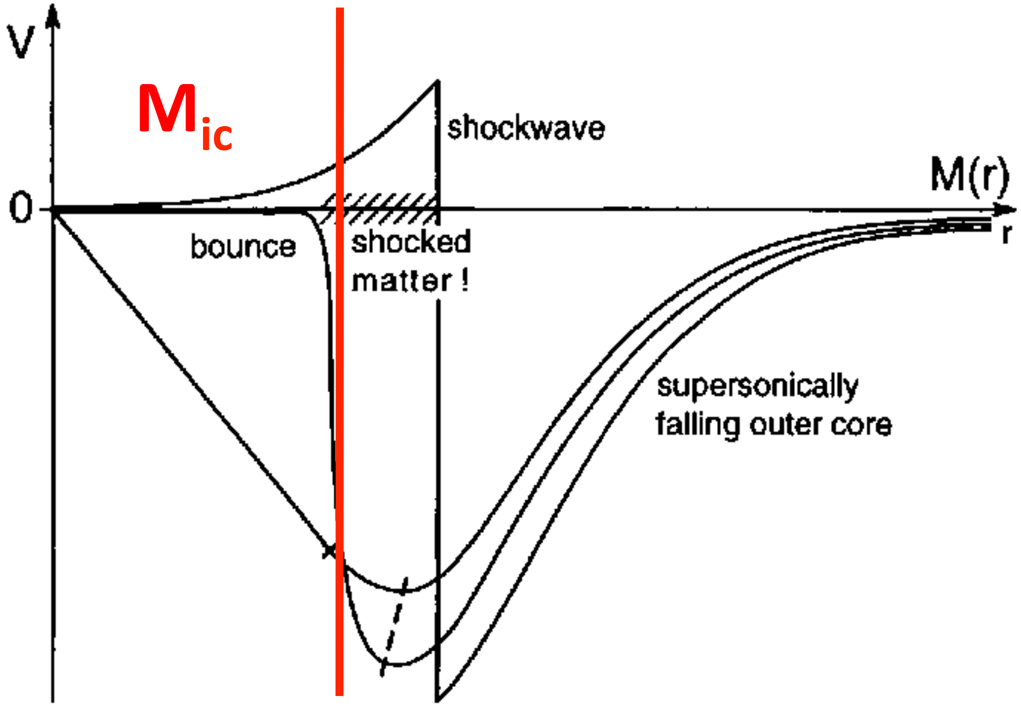


Note: EOS unrealistically soft; ruled out by 2-M_{Sun} NS' Demorest+'10

Stellar Collapse: Inner Core Mass

Why worry about M_{ic}?

Bethe 1990!!!



- M_{ic} is the amount of matter dynamically relevant in bounce.
- M_{ic} sets kinetic energy imparted to the shock.
- M_{ic} (and IC radius) sets the angular momentum that can be dynamically relevant.
- Sets mass cut for material that the shock needs to go through.
- M_{ic} ~0.5 M_{SUN} can easily stabilized by nuclear EOS.
  -> **No "prompt" Black Hole formation.**
- M_{ic} sets the mass that must be accreted (before explosion?) to make a canonical 1.4 M_{SUN} neutron star.

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