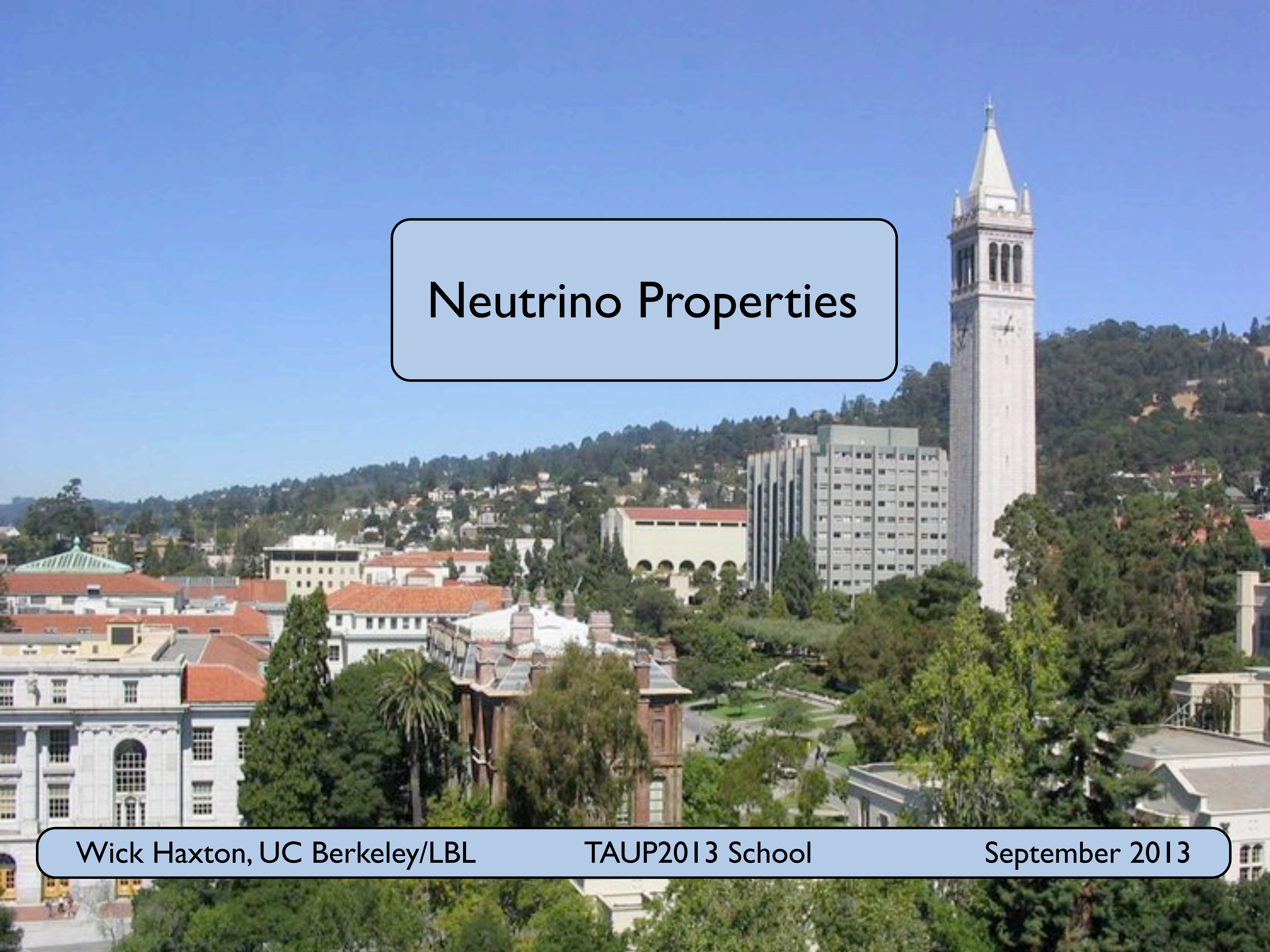


Neutrino Properties

Wick Haxton, UC Berkeley/LBL TAUP2013 School September 2013

The plan:

- weak interaction & currents: history
- lepton number
- Dirac and Majorana masses: double beta decay
- neutrino oscillations
- the MSW mechanism
- kinematic tests of neutrino mass
- CP violation
- the many open questions

Liebe Radioaktive Damen and Herren.....

- Experiments on radioactive nuclei had, by the mid-1920s, demonstrated that the positrons emitted in beta decay carried off only about half of the energy expected to be released in the nuclear decay

- Speculations included Niels Bohr's suggestion that mass/energy equivalence might not hold in the new "quantum mechanics;" and Chadwick's suggestion that perhaps some unobserved and unmeasured radiation accompanied the positron

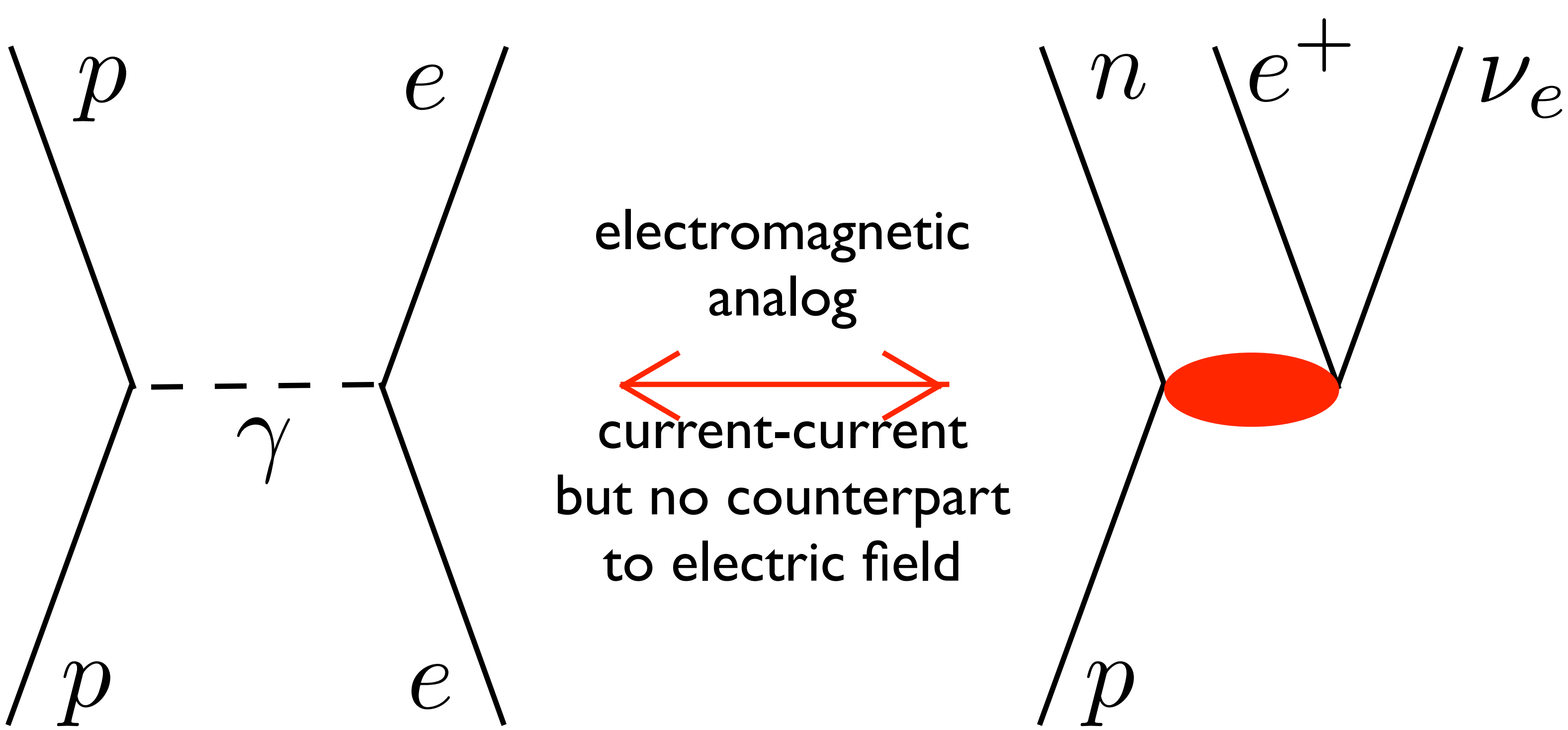
- In 1930 Pauli hypothesized that an unobserved neutral, spin-1/2 "neutron" accounted for the apparent anomaly -- a new particle with mass < 1% that of the proton, the ν

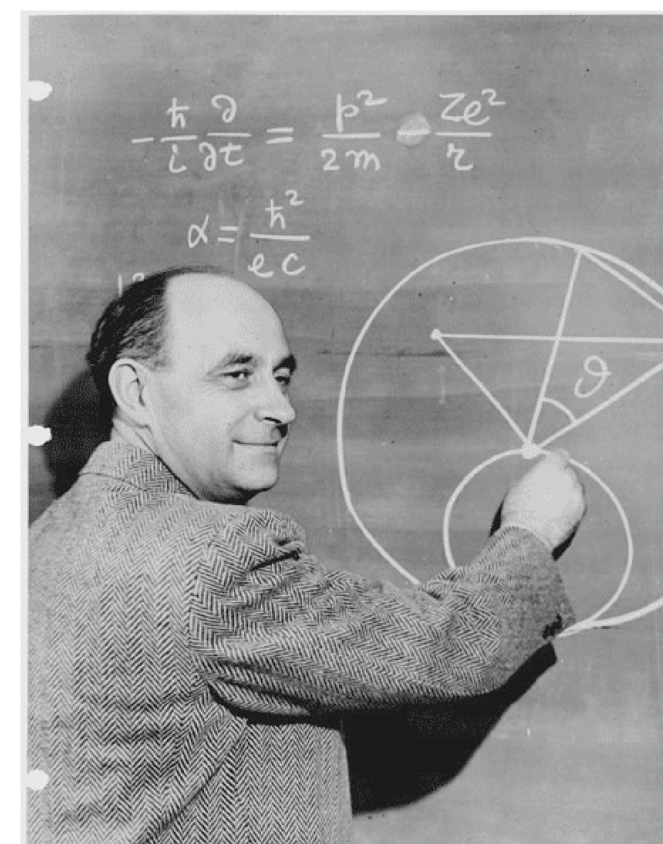
"... a genius, comparable perhaps only to Einstein himself" N. Bohr

"I have done a terrible thing. I have postulated a particle that cannot be detected."

- Pauli viewed the ν as an atomic constituent -- knocked out in the β decay process
- Chadwick's 1932 discovery of (today's) neutron
- prompted Fermi to propose (1934)

1933 7th Solvay Conference: Pauli's first public presentation of the neutrino





Fermi

We can look at this from a slightly more modern view: introduce isospin to distinguish otherwise nearly identical p,n

$$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \tau_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \tau_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\frac{1+\tau_3}{2}\, p = +p \quad \frac{1+\tau_3}{2}\, n = 0n \quad \tau_+\, n = p \quad \tau_-\, p = n$$

so e-neutron or e-proton interaction vs. weak interaction

$$[e]\ \frac{1}{r}\ \left[e\frac{1+\tau_3}{2}\right] \qquad \Leftrightarrow \qquad [e]\ \frac{\delta(\vec{r})}{M^2}\ \left[\mp\frac{1}{\sqrt{2}}e\tau_\pm\right]$$

E&M: $\rho^S + \rho^{V(0)}$ weak $\rho^{V(\pm)}$

makes sense: Fermi used the "missing" components of isovector charge

Fermi recognized that Lorentz invariance meant that this relation must extend to currents (moving charges), $\rho \to j^\mu = (\rho, \vec{j}) = e(1, \vec{p}/M_N)$

$$j^{E\&M} = j_\mu^{V;S} + j_\mu^{V;V(0)} \qquad \Leftrightarrow \qquad j^{Weak} = j_\mu^{V;V(\pm)}$$

Weak current a space-spin vector and an isospin isovector:
E&M and the weak interaction made use of all three isospin components of the vector hadronic current: a step toward unification!

Then:

THE

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G. GAMOW AND E. TELLER, *George Washington University, Washington D. C.*

(Received March 28, 1936)

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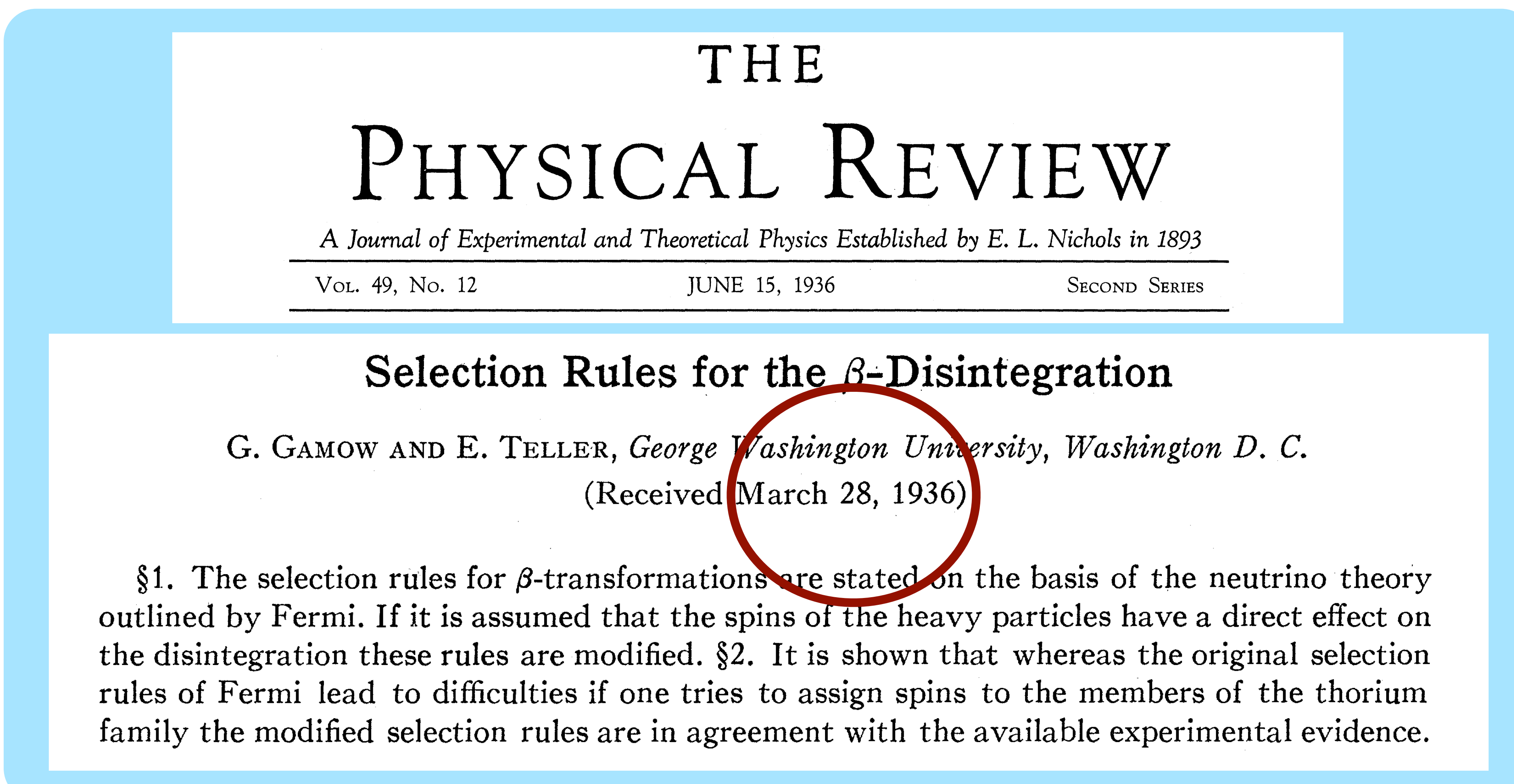

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Fermi's β-decay ↔ electromagnetism analogy ↔ vector weak current ⇒

	$\mu = 0$	$\mu = 1,2,3$
$j_\mu^{weak} = j_\mu^{V;V_\pm}$	$1\ \tau_\pm$	$\vec{p}/m_N\ \tau_\pm$

Fermi's relativistic correction, noted by G and T

⇒ selection rules for "allowed" decays of

ΔJ = 0 Δπ = 0, e.g., $0^+ \rightarrow 0^+$ decays

with relativistic corrections

ΔJ = 0, ±1 (but no 0→0) Δπ = 1, e.g., $1^- \rightarrow 0^+$ decays: suppressed by $(v/c)^2$ in transition probabilities

GT added an axial contribution to Fermi's interaction

	$\mu = 0$	$\mu = 1,2,3$
$j_\mu^{weak} = j_\mu^{V;V_\pm}$	$1 \ \tau_\pm$	$\vec{p}/m_N \ \tau_\pm$
$+j_\mu^{A;V_\pm}$	$\vec{\sigma} \cdot \vec{p}/m_N \ \tau_\pm$	$\vec{\sigma} \ \tau_\pm$

ordinary vector

$\sim \vec{r} \times \vec{p}$
carries opposite parity
pseudo- or axial-vector

So that one could obtain in lowest order (allowed)

Fermi: $\Delta J = 0$ $\Delta\pi = 0$, e.g., $0^+ \rightarrow 0^+$ decays and
Gamow-Teller: $\Delta J = 0, \pm 1$ (but no $0 \rightarrow 0$) $\Delta\pi = 0$, e.g., $1^+ \rightarrow 0^+$

"Either the matrix element M_1 or the matrix element M_2 or finally a linear combination of M_1 and M_2 will have to be used to calculate the probabilities of the β-disintegrations. If the third possibility is the correct one, and the two coefficients in the linear combination have the same order of magnitude, then all transitions [satisfying the selection rules] would now [be strong allowed ones]"

- They had deduced the correct rate for beta decay

$$\omega \sim |\langle 1 \rangle|^2 + g_A^2 |\langle \vec{\sigma} \rangle|^2$$

- They obtained this result by generalizing Fermi's interaction into a sum of four-fermion interactions

$$j_\mu^{lep\ V;\mp} j_\mu^{nucl\ V;\pm} \Leftrightarrow j_\mu^{lep\ V;\mp} j_\mu^{nucl\ V;\pm} + j_\mu^{lep\ A;\mp} j_\mu^{nucl\ A;\pm}$$

- But failed to comment on a second possible generalization

$$j_\mu^{lep\ V;\mp} j_\mu^{nucl\ V;\pm} = \left(j_\mu^{lep\ V;\mp} - j_\mu^{lep\ A;\mp}\right) \left(j_\mu^{nucl\ V;\pm} - j_\mu^{nucl\ A;\pm}\right)$$

This alternative gives the same β-decay formula, but implies

- weak interaction is parity violating !
- the neutrino has a definite helicity

20 years before PNC, 35 years before the SM & neutral weak currents

- in fact the correct low-energy effective Hamiltonian for weak interactions is

$$H_{\rm weak} \sim \frac{G_F}{\sqrt{2}} \left(j_\mu^{lep\ V;\mp} - j_\mu^{lep\ A;\mp}\right) \left(j_\mu^{nucl\ V;\pm} - j_\mu^{nucl\ A;\pm}\right)$$

$$G_F \sim \frac{e^2}{M_W^2}$$

not what GT chose.

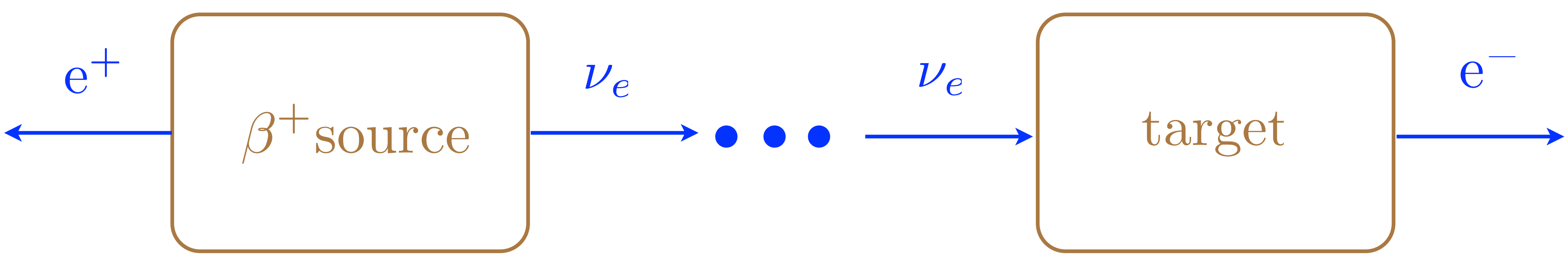
This unwarranted fondness for parity nonconservation led to other unfounded conclusions as well...

Beta decay and lepton number

C (actually CPT) guarantees that each particle has an antiparticle --
this operation reverses "charges," the additively conserved quantum nos.

$e^- \to e^+$ clearly particle and antiparticle are distinct
but what about the neutrino? is the antiparticle distinct from particle?

so we do an experiment:



this defines the ν_e

which is then found to produce: e^-

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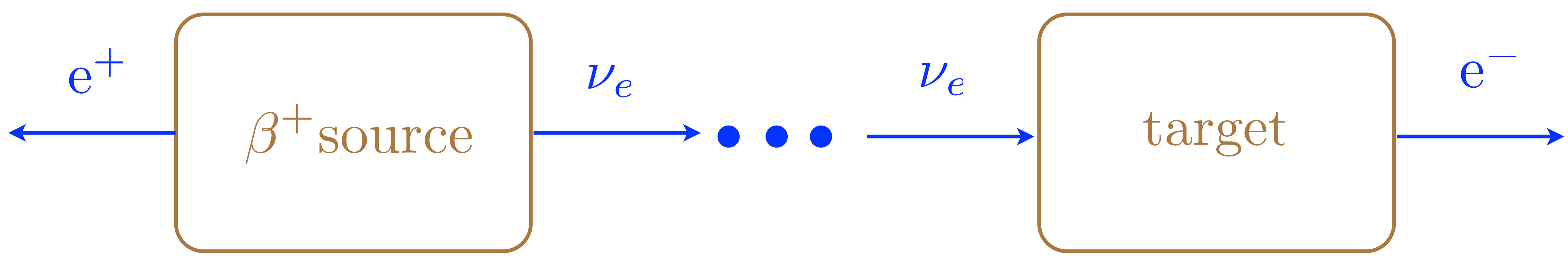
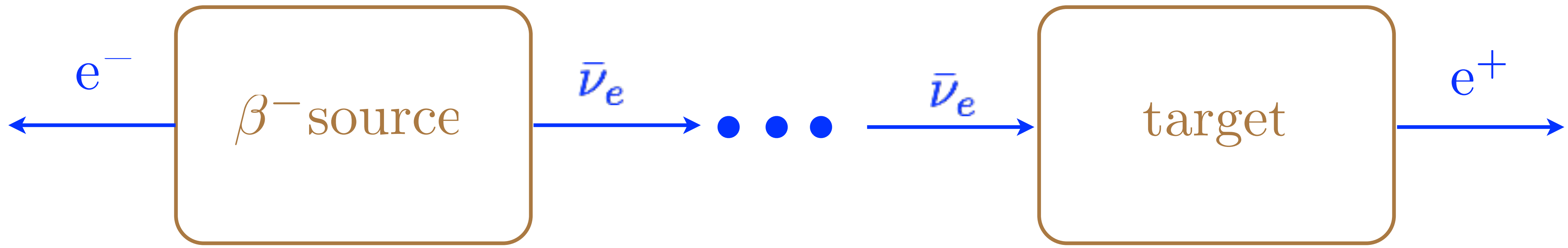
Neutrinos less constrained

so we do an experiment:



this defines the ν_e which is then found to produce: e^-

and a second one:



this defines the $\overline{\nu}_e$

which is then found to produce: e^+

- with these definitions of the ν_e and $\bar{\nu}_e$, they appear operationally distinct, producing different final states

- introduce a "charge" to distinguish the neutrino states and to define the allowed reactions, l_e, which we require to be additively conserved

$$\sum_{in} l_e = \sum_{out} l_e$$

lepton	l_e
e^-	$+1$
e^+	-1
ν_e	$+1$
$\bar{\nu}_e$	-1

- so this would

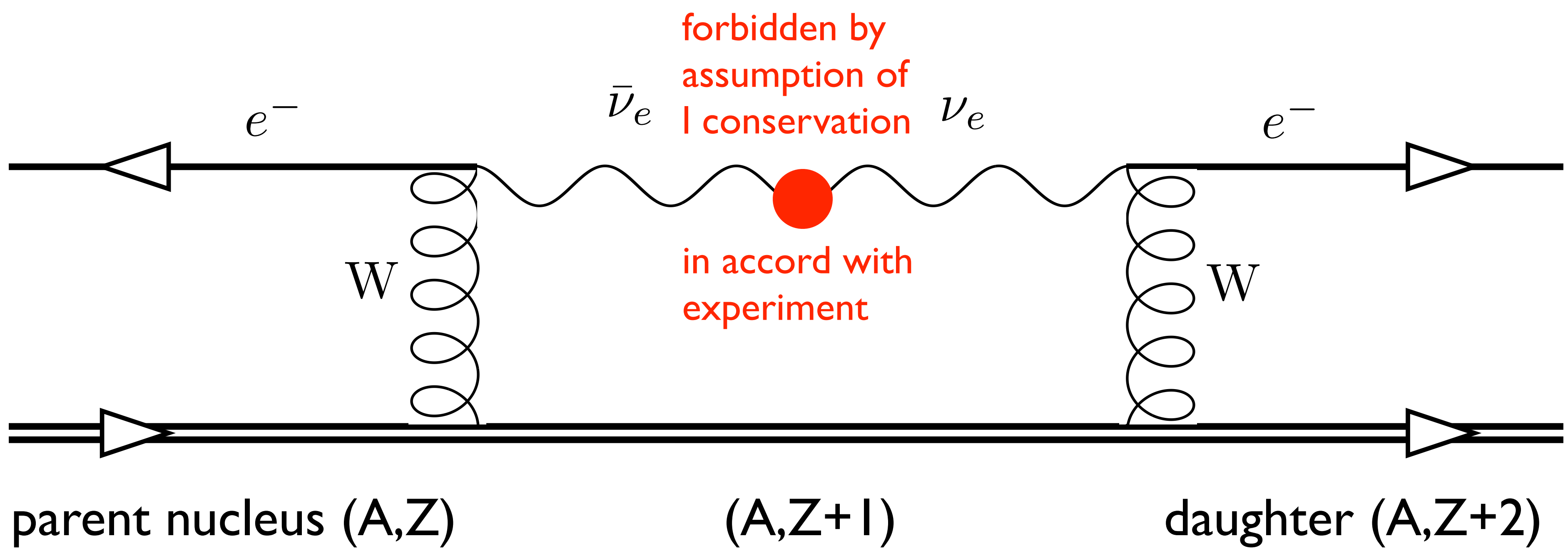
$$\text{allow } \nu_e + n \rightarrow e^- + p \quad \text{but not} \quad \bar{\nu}_e + n \rightarrow e^- + p$$
$$\text{allow } \bar{\nu}_e + p \rightarrow e^+ + n \quad \text{but not} \quad \nu_e + p \rightarrow e^+ + n \quad \text{and so on}$$

- can generalize for ν_μ and ν_τ: conservation of separate lepton number

- or can consider a weaker conservation law of total lepton number

$$\sum_{in} l_e + l_\mu + l_\tau = \sum_{out} l_e + l_\mu + l_\tau$$

These experiments are done virtually in neutrinoless $\beta\beta$ decay (Gabriel)



- This process is produces a final state with two electrons, starting from an initial state with none -- lepton number violating, and thus forbidden by the rules just "derived" in our gedanken experiments

- Such "neutrinoless double beta decay" was not seen, and this in fact was how the experiments just described were actually "done"

- the above argument makes an implicit assumption -- the same one that Gamow and Teller made

Neutrino helicity

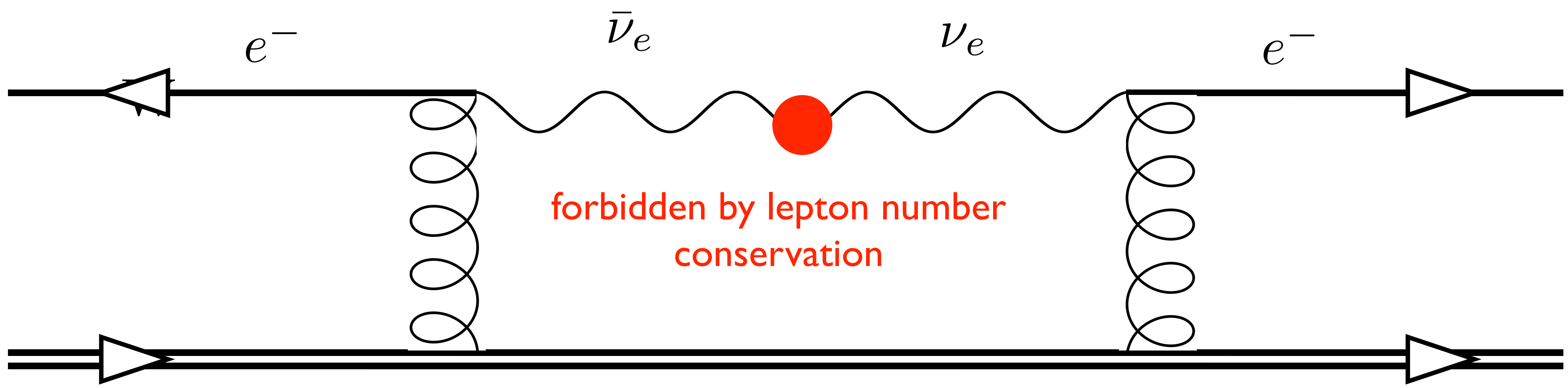
- parity was used early in the 1920s to classify atomic wave functions and atomic transitions (E&M): regarded as an important symmetry by GT, clearly, and also in early studies of double beta decay

- in 1956 Lee and Yang considered the tau-theta puzzle, the apparent existence of a pair of equal-mass mesons, one of which has negative parity and decays into three pions, the other with positive parity and decaying into two pions: observed that the experimental support for parity conservation was limited to the strong and E&M interactions

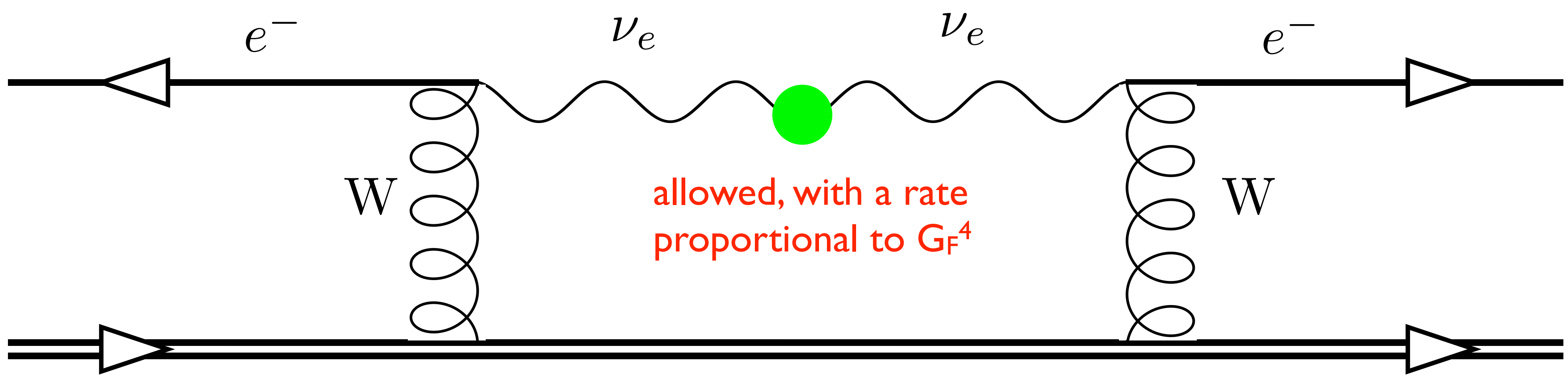
- parity violation demonstrated by
    - Wu et al: angular asymmetry of βs from the decay of polarized ^{60}Co
    - Garwin, Lederman, and Weinrich: large μ polarization in π β-decay from the angular distribution of μ-decay electrons
    - Goldhaber-Grodzins-Sunyar: β-decay νs are left-handed

to the extent we can measure, PNC is maximal: V-A

If there is a conserved lepton number

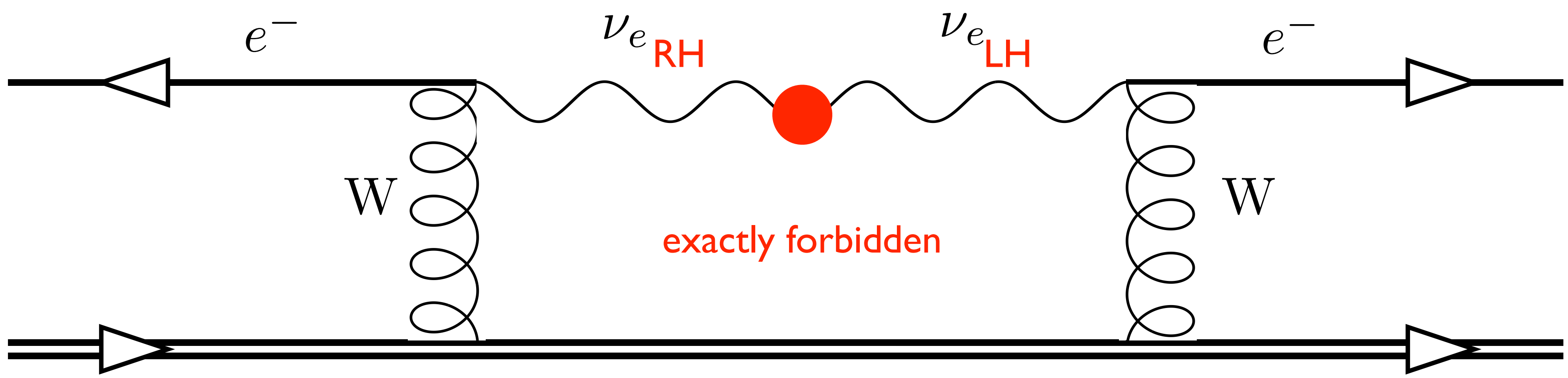


Remove the restriction of an additively conserved lepton number

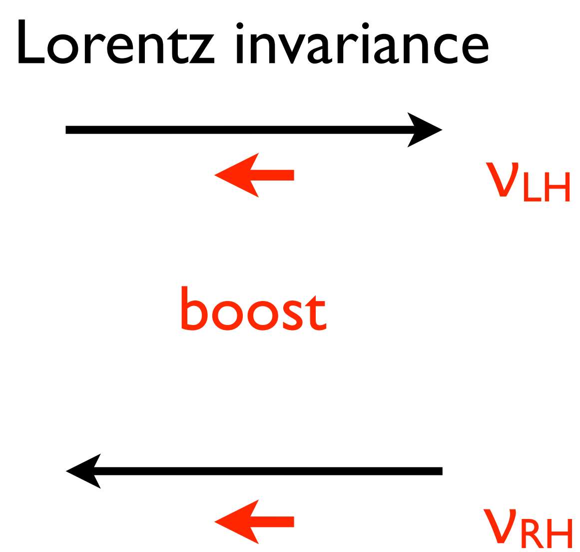


conflicts with experimental
upper bounds on rates

Account for absence of $\beta\beta$ decay by the exact V-A nature of weak currents



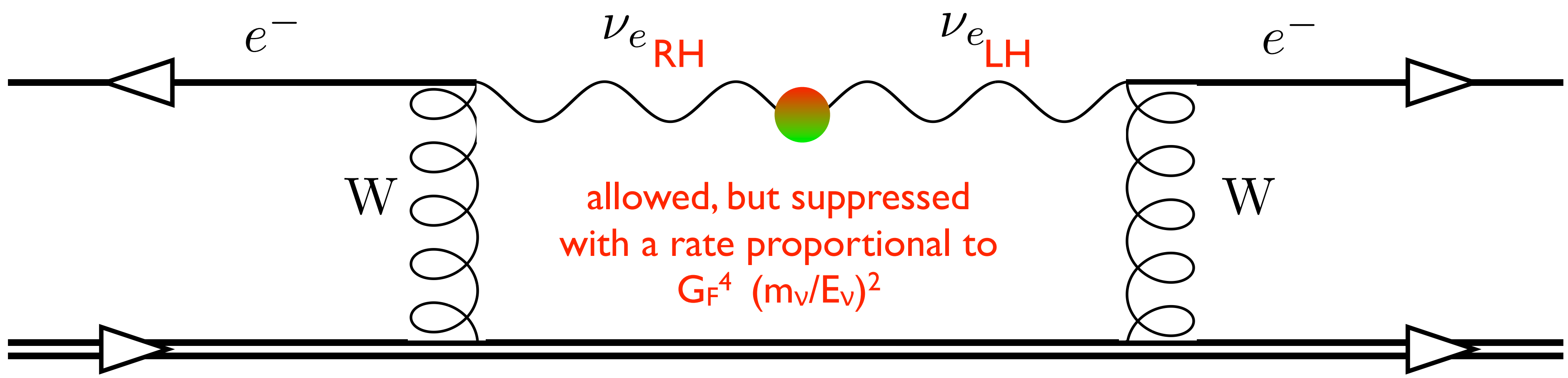
What we mean by a LHed neutrino is



but if massive

so connected with mass -- handedness cannot be exact for a massive ν

So if neutrinos have masses, the rate is not forbidden, but only suppressed

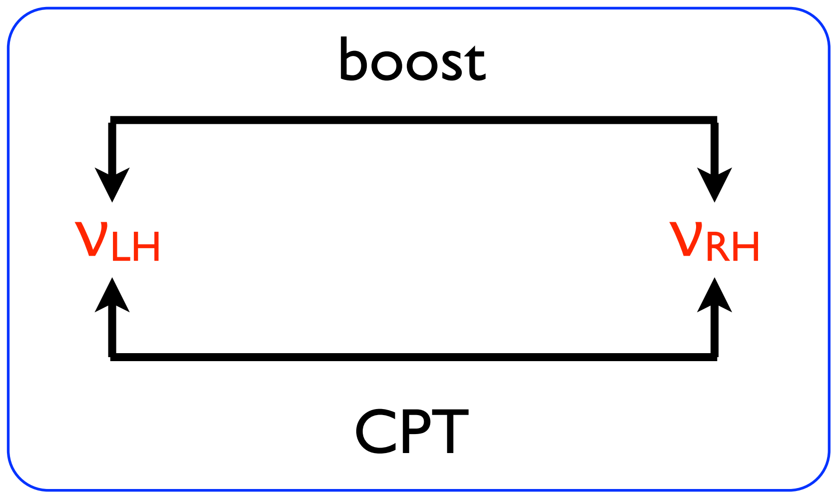


neutrino mass restores $\beta\beta$ decay as a definitive test of lepton number violation, at the cost of a rate proportional to m_ν/E_ν where $E_\nu \sim 1/R_{nuclear}$

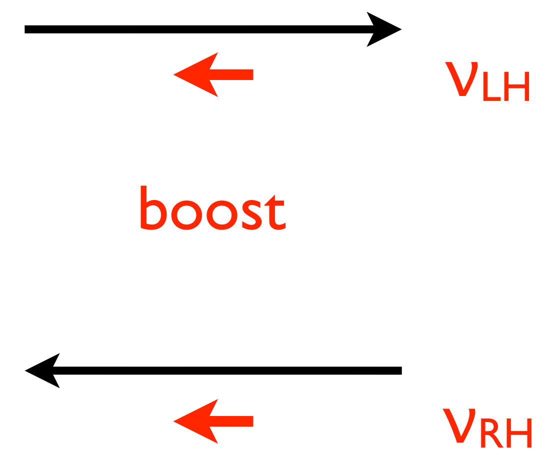
the neutrino mass plays two roles, breaking the γ_5 invariance and likely providing the source of the lepton number violation

Massive neutrino descriptions

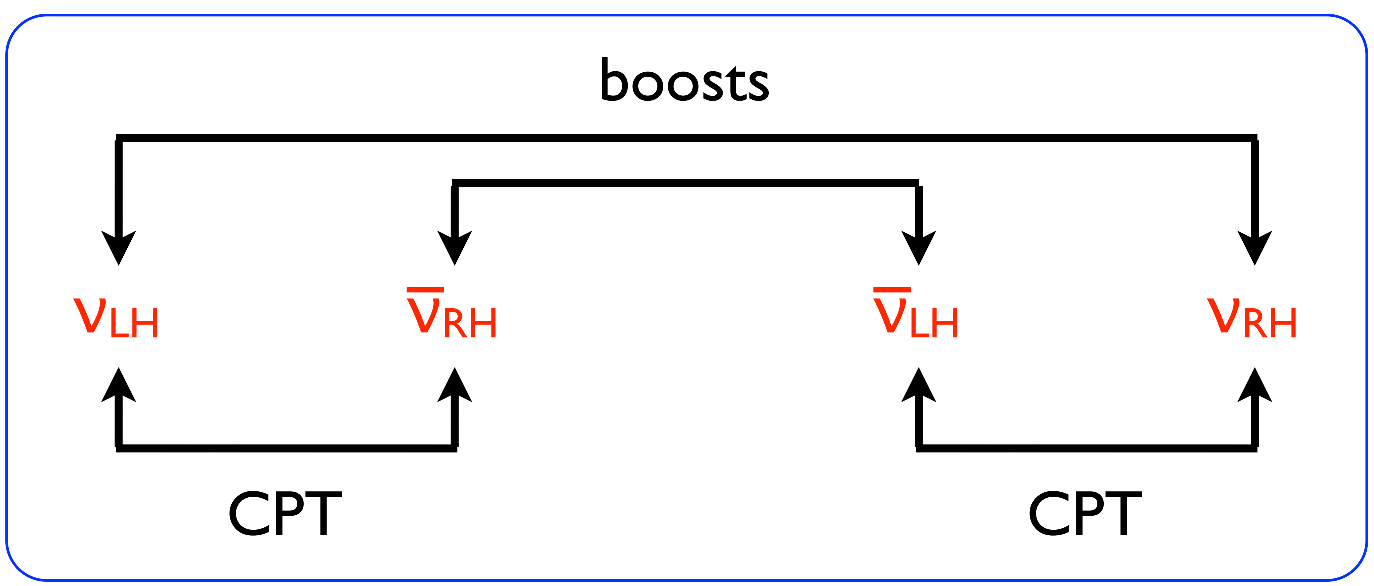
Majorana:



Lorentz invariance



Dirac:



or some linear combination of the two: what is not forbidden is required

Let's see the mass consequences: start with the Dirac eq., project out

$$\psi_{R/L} = \frac{1}{2}(1 \pm \gamma_5)\psi] \qquad C\ \psi_{R/L}\ C^{-1} = \psi^c_{R/L}$$

Allow for multiple flavors and flavor mixing

$$L_m(x) \sim m_D \bar{\psi}(x)\psi(x) \Rightarrow M_D \bar{\Psi}(x)\Psi(x) \qquad \Psi_L \equiv \begin{pmatrix} \Psi^e_L \\ \Psi^\mu_L \\ \Psi^\tau_L \end{pmatrix}$$

Gives a 4n by 4n matrix, n the number of generations

$$(\bar{\Psi}^c_L, \bar{\Psi}_R, \bar{\Psi}_L, \bar{\Psi}^c_R) \begin{pmatrix} 0 & 0 & & M^T_D \\ 0 & 0 & M_D & \\ & M^\dagger_D & 0 & 0 \\ M^*_D & & 0 & 0 \end{pmatrix} \begin{pmatrix} \Psi^c_L \\ \Psi_R \\ \Psi_L \\ \Psi^c_R \end{pmatrix}$$

Observe that the handedness allows an additional generalization

$$L_m(x) \Rightarrow M_D \bar{\Psi}(x)\Psi(x) + (\bar{\Psi}^c_L(x) M_L \Psi_L(x) + \bar{\Psi}^c_R(x) M_R \Psi_R(x) + h.c.)$$

to give the more general matrix

$$(\bar{\Psi}^c_L, \bar{\Psi}_R, \bar{\Psi}_L, \bar{\Psi}^c_R) \begin{pmatrix} 0 & 0 & M_L & M_D^T \\ 0 & 0 & M_D & M_R^\dagger \\ M_L^\dagger & M_D^\dagger & 0 & 0 \\ M_D^* & M_R & 0 & 0 \end{pmatrix} \begin{pmatrix} \Psi^c_L \\ \Psi_R \\ \Psi_L \\ \Psi^c_R \end{pmatrix}$$

which has a number of interesting properties:

- the eigenvectors are two-component Majorana spinors: 2n of these
- the introduction of M_L, M_R breaks the global invariance $\Psi \rightarrow e^{i\alpha}\Psi$ associated with a conserved lepton number

- Dirac limit: the removal of M_L, M_R makes the eigenvalues pairwise degenerate: two two-component spinors of opposite CP can be patched together to form a four-component Dirac spinor -- so one gets n of these

- The mass parameter tested in $\beta\beta$ decay is

$$\sum_{\mathbf{i=1}}^{\mathbf{2n}} \mathbf{U^L_{ei} U^L_{ei}} \lambda_{\mathbf{i}}^{\mathbf{CP}} \mathbf{m}_\nu^{\mathbf{i}}$$

  where CP conservation is assumed: here $\mathbf{U_{ei}}$ is real, $\mathbf{m}_\nu^{\mathbf{i}} \geq \mathbf{0}$, and $\lambda_{\mathbf{i}}^{\mathbf{CP}}$ is the ith's neutrino's CP eigenvalue. So mass terms can interfere according to their relative CP -- and interfere totally in the Dirac limit

- And in fact there can be CP violation: these phases can be complex

- The restriction against masses in the SM are quite artificial: Dirac masses cannot be constructed without a RHed neutrino field, and there is none; one can construct M_L but the SM coupling would be $\sim \phi^2/M_{\rm new}$ (the simplest "effective operator" one can introduce in the SM)

- But this is very attractive...

because it suggested the seesaw explanation of the anomalous ν mass scale

- give the ν an M_D typical of other SM fermions
- take $M_L \sim 0$, in accord with $\beta\beta$ decay
- assume $M_R >> M_D$ as we have not found new RHed physics at low E

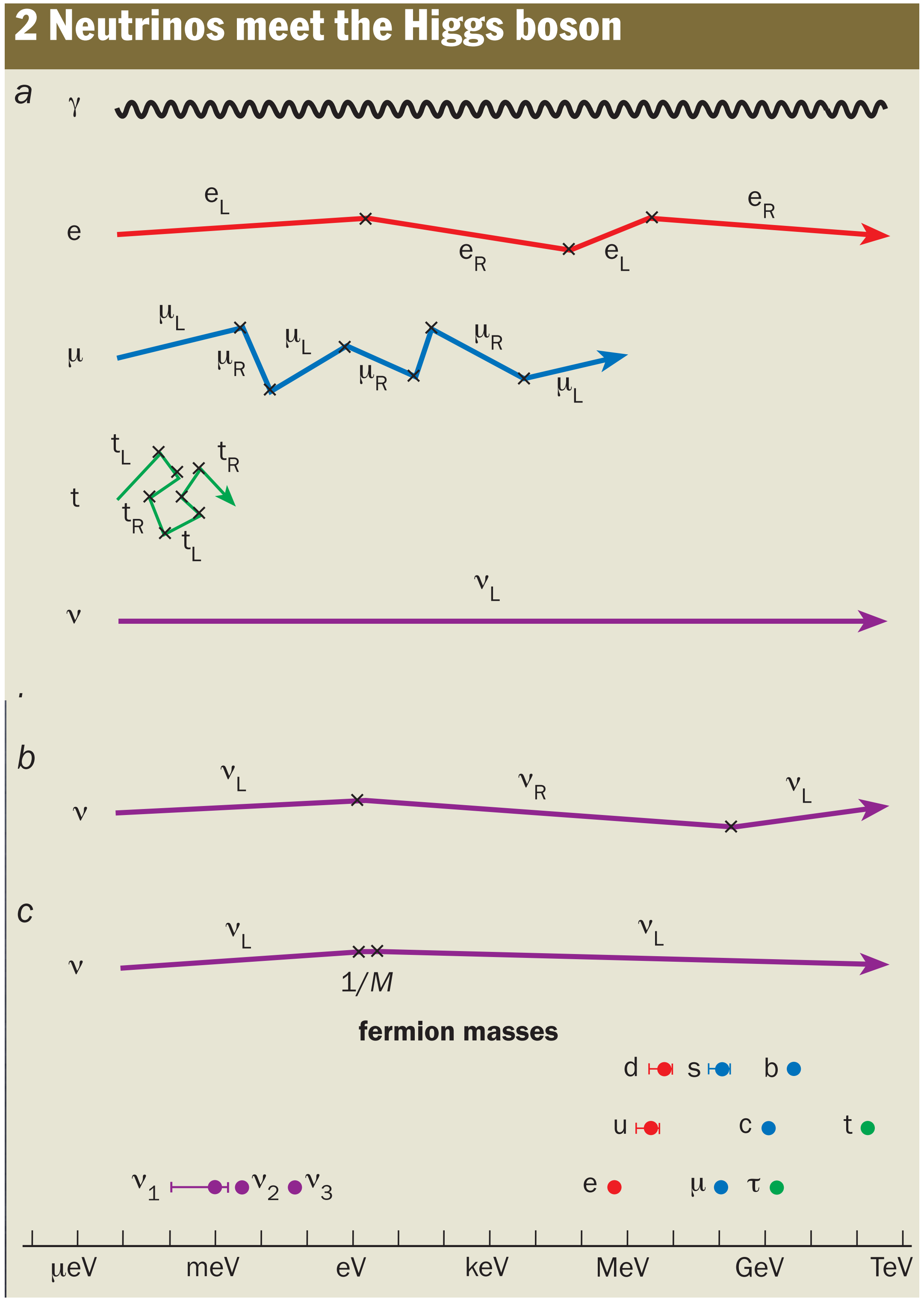
$$\begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \Rightarrow \; m_\nu^{\text{light}} \sim m_D \left(\frac{m_D}{m_R}\right)$$

- take $m_\nu \sim \sqrt{m^2{}_{23}} \sim 0.05$ eV and $m_D \sim m_{top} \sim 180$ GeV

$$\Rightarrow m_R \sim 0.3 \times 10^{15} \text{ GeV} \quad !$$

Effectively the additional flexibility available in constructing ν mass, the fact both Dirac and Majorana terms are allowed, can be exploited to explain the anomalous scale of neutrino mass

Murayama's ν mass cartoon



standard model masses:
scattering off the Higgs field

light Dirac neutrino

LHed Majorana neutrino:
entirely new

← COMBINE:
the anomalous ν mass scale

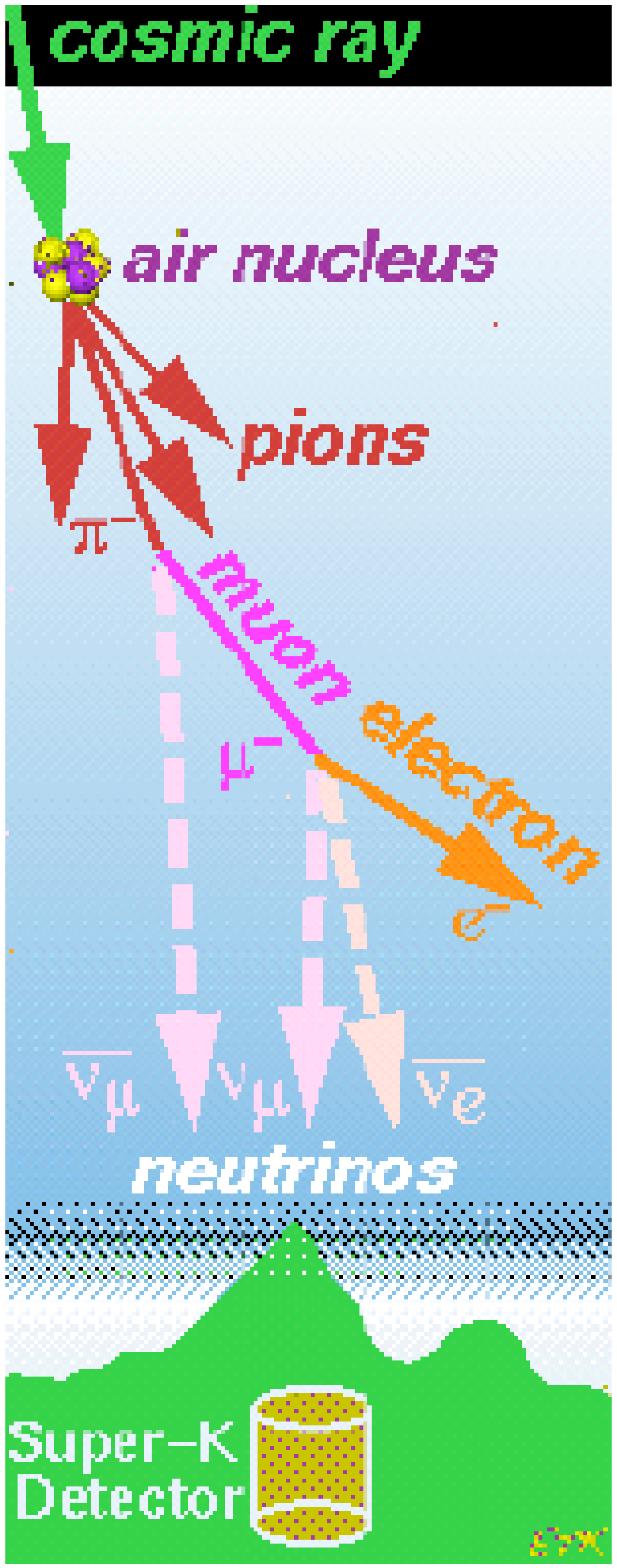
So we see neutrino masses are exceedingly interesting

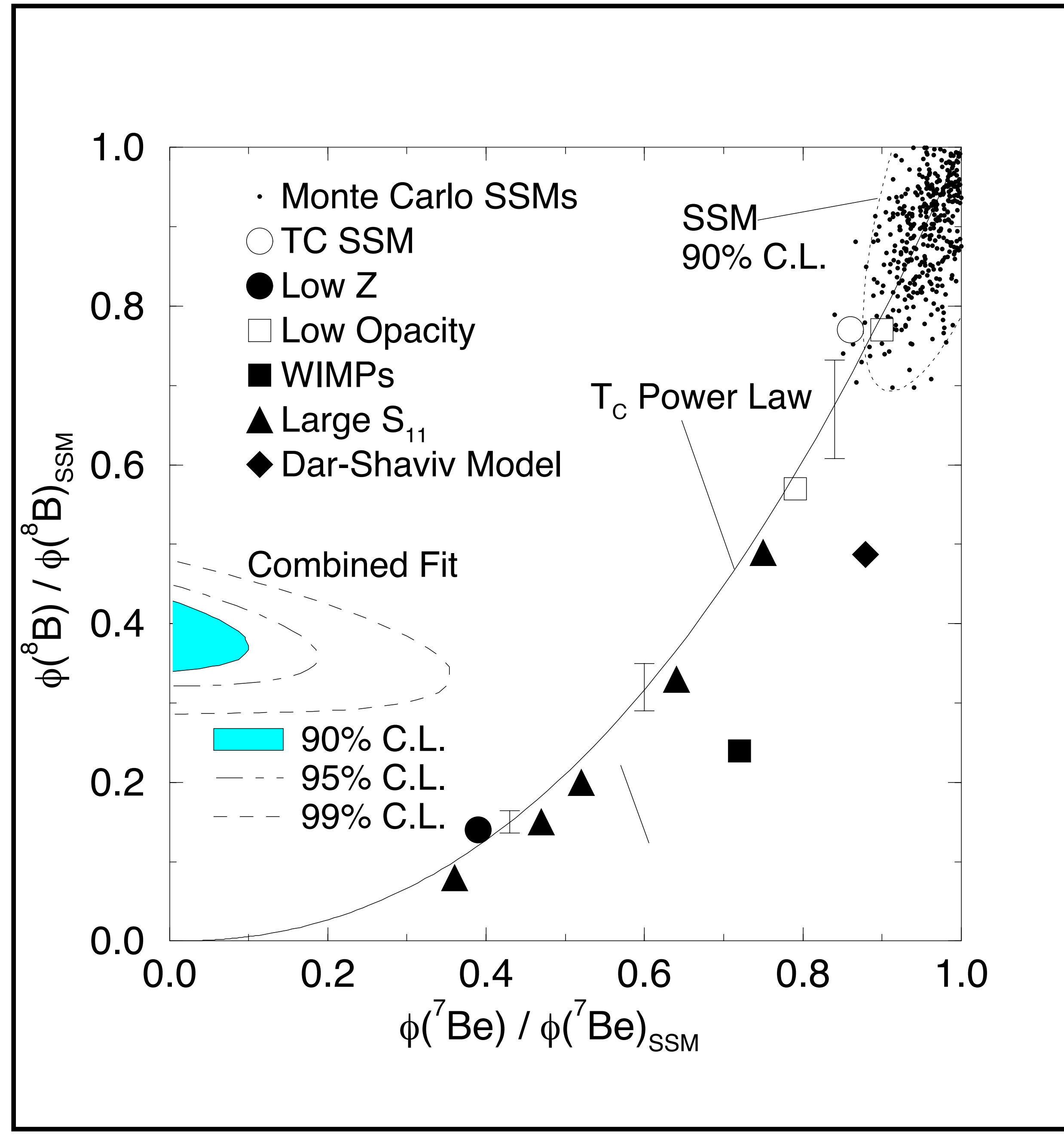
- a new mass mechanism, BSM
- a possible resolution of the anomalous scales of SM particle masses: necessary to ever building a model that accounts for the pattern of masses we see in nature
- potentially connected to energies 11 orders of magnitude beyond our reach

Thus we look for other ways of testing mass

- neutrino oscillations
- direct mass tests

In both cases astrophysics provided the initial breakthroughs





Learned that νs are massive from atmospheric and solar experiments: oscillations

Vacuum flavor oscillations: mass and weak eigenstates

flavor states	$\|\nu_e>$	$\leftrightarrow$	$\|\nu_L>$	m_L	mass states
	$\|\nu_\mu>$		$\|\nu_H>$	m_H	

Noncoincident bases ⇒ oscillations down stream:

$$\begin{aligned} |v_e> &= \cos\theta|\nu_L> + \sin\theta|\nu_H> \\ |v_\mu> &= -\sin\theta|\nu_L> + \cos\theta|\nu_H> \end{aligned}$$

$$\begin{aligned} |\nu_e^k> &= |\nu^k(x=0,t=0)> \qquad E^2 = k^2 + m_i^2 \\ |\nu^k(x\sim ct,t)> &= e^{ikx}\left[e^{-iE_Lt}\cos\theta|\nu_L> + e^{-iE_Ht}\sin\theta|\nu_H>\right] \\ |<\nu_\mu|\nu^k(t)>|^2 &= \sin^2 2\theta \sin^2\left(\frac{\delta m^2}{4E}t\right), \qquad \delta m^2 = m_H^2 - m_L^2 \end{aligned}$$

ν_μ appearance downstream ⇔ vacuum oscillations

(some cheating here: wave packets)

Original suggestion of vacuum neutrino oscillations came from Pontecorvo, who was seeking to solve a problem...

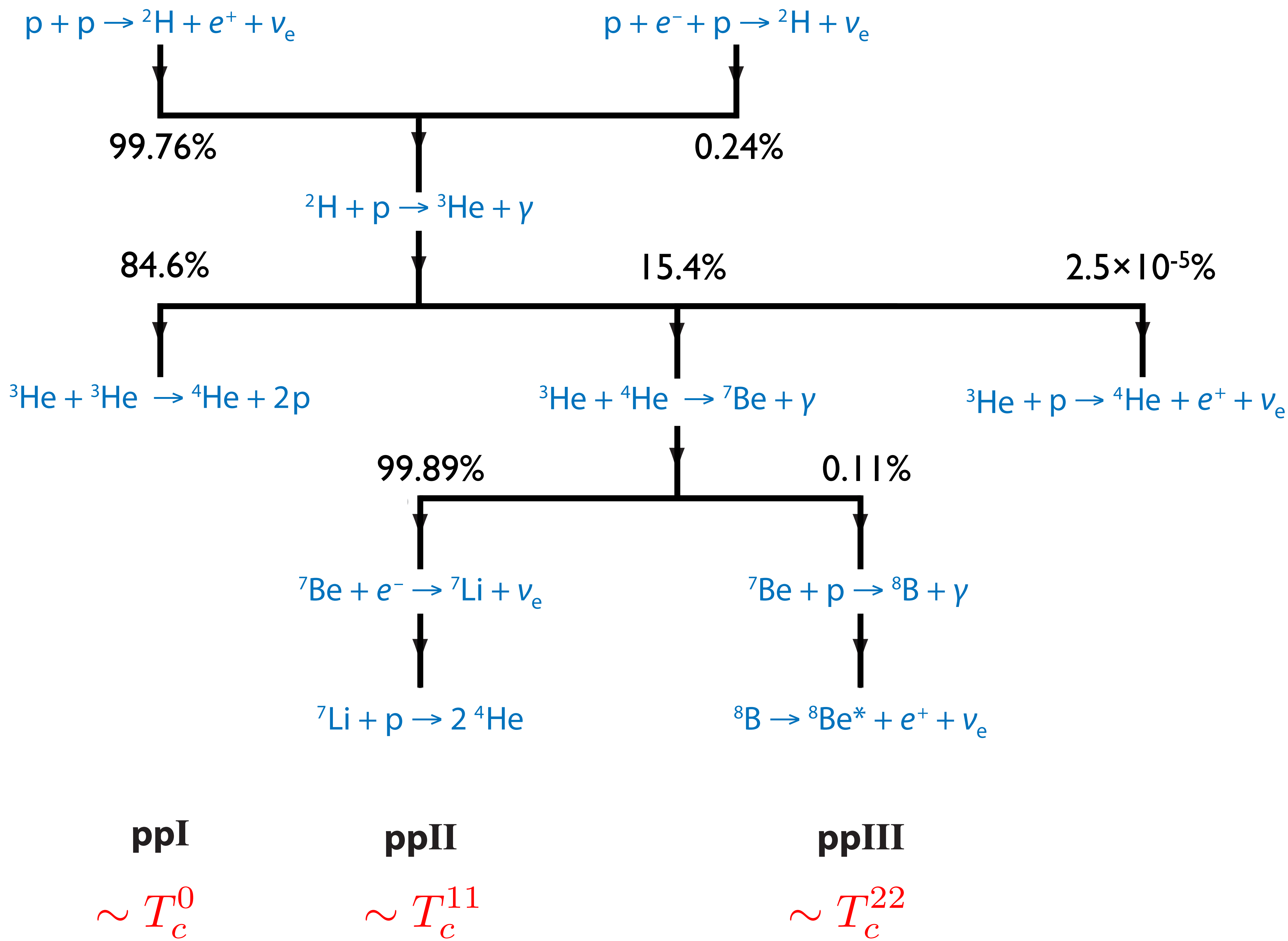
The number of $\nu_e s$ coming from the Sun was about 1/3rd that expected

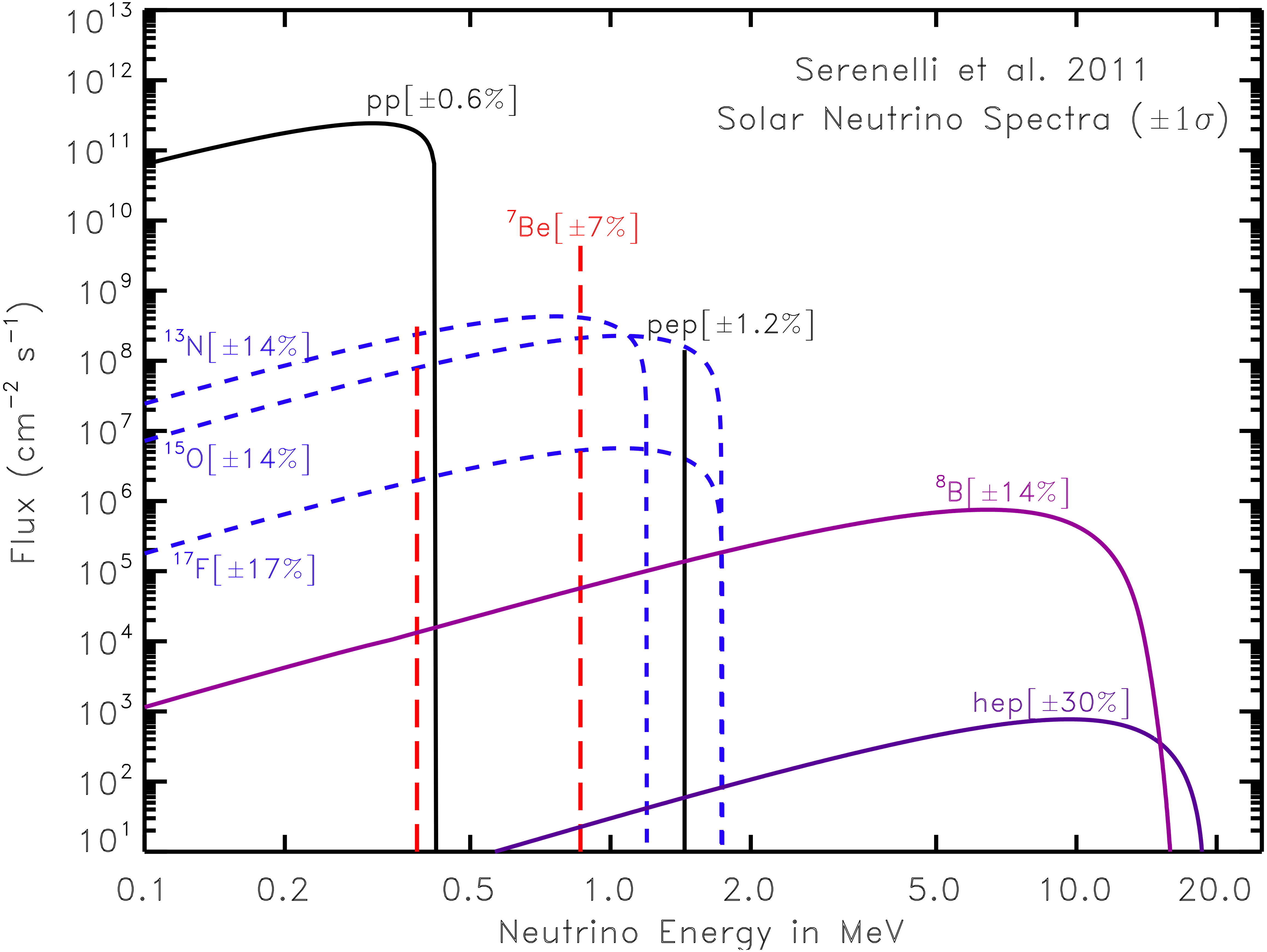
But Pontecorvo's suggestion seemed a long-shot

$$|\langle \nu_e | \nu(t) \rangle|^2 \sim 1 - \sin^2 2\theta \sin^2 \frac{\delta m^2}{4E} t \to 1 - \frac{1}{2} \sin^2 2\theta$$

Even a maximal mixing angle gave only a factor of 1/2.

The only mixing angles known were among the quarks, and they are small.

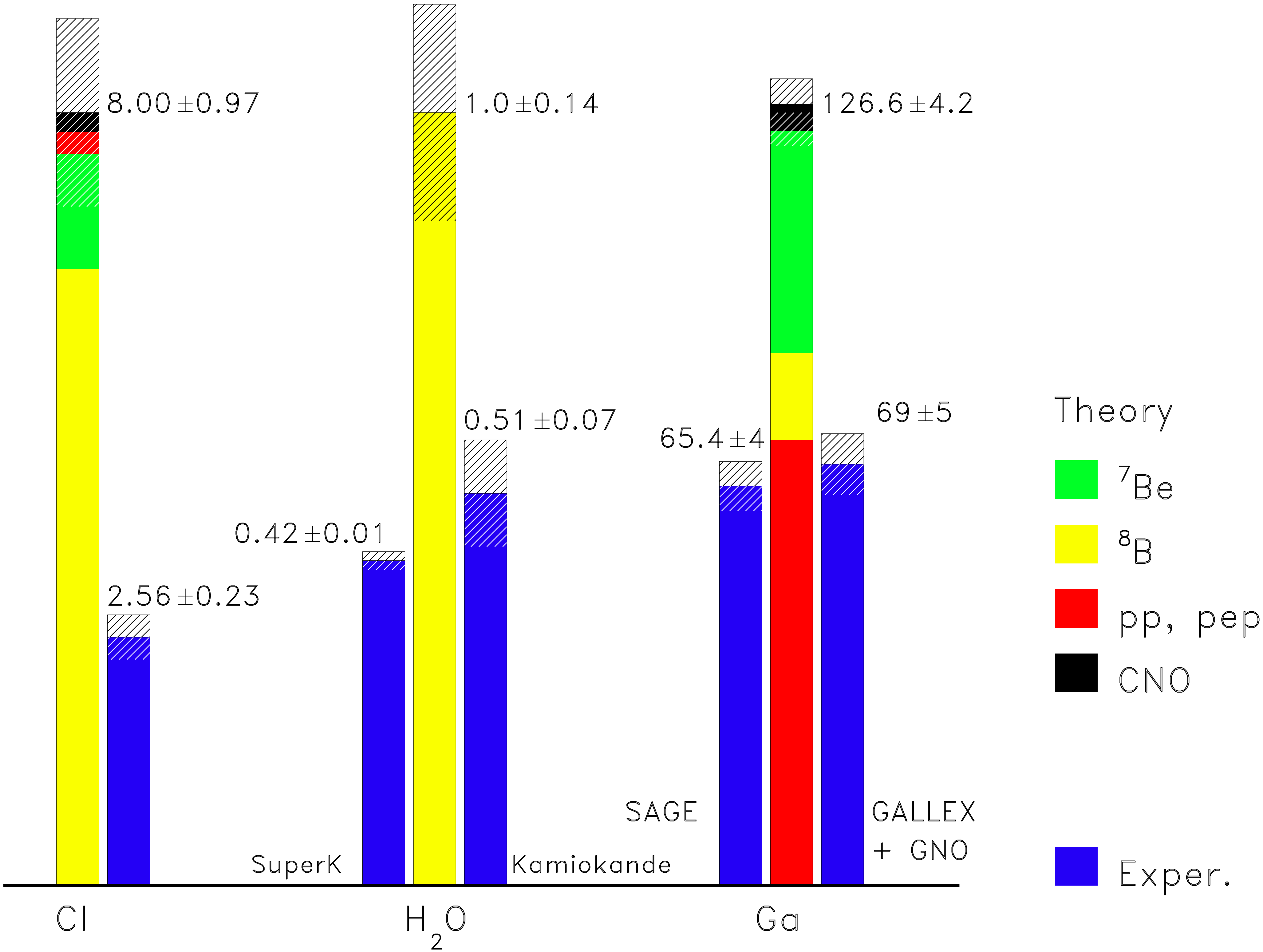

p + p → ²H + e⁺ + ν_e
p + e⁻ + p → ²H + ν_e
99.76%
0.24%
²H + p → ³He + γ
84.6%
15.4%
2.5×10⁻⁵%
³He + ³He → ⁴He + 2p
³He + ⁴He → ⁷Be + γ
³He + p → ⁴He + e⁺ + ν_e
99.89%
0.11%
⁷Be + e⁻ → ⁷Li + ν_e
⁷Be + p → ⁸B + γ
⁷Li + p → 2 ⁴He
⁸B → ⁸Be* + e⁺ + ν_e
ppI
ppII
ppIII
∼ T_c^0
∼ T_c^11
∼ T_c^22



Serenelli et al. 2011
Solar Neutrino Spectra (±1σ)
pp[±0.6%]
7Be[±7%]
pep[±1.2%]
13N[±14%]
15O[±14%]
17F[±17%]
8B[±14%]
hep[±30%]
Flux (cm−2 s−1)
Neutrino Energy in MeV
0.1
0.2
0.5
1.0
2.0
5.0
10.0
20.0
10^13
10^12
10^11
10^10
10^9
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1







8.00 ±0.97
2.56 ±0.23
1.0 ±0.14
0.42 ±0.01
0.51 ±0.07
126.6 ±4.2
65.4 ±4
69 ±5
SuperK
Kamiokande
SAGE
GALLEX
+ GNO
Cl
H2O
Ga
Theory
7Be
8B
pp, pep
CNO
Exper.

Let's go back to vacuum oscillations, slightly generalizing our initial state

$$|\nu(0)\rangle \;\rightarrow\; a_e(0)|\nu_e\rangle + a_\mu(0)|\nu_\mu\rangle$$

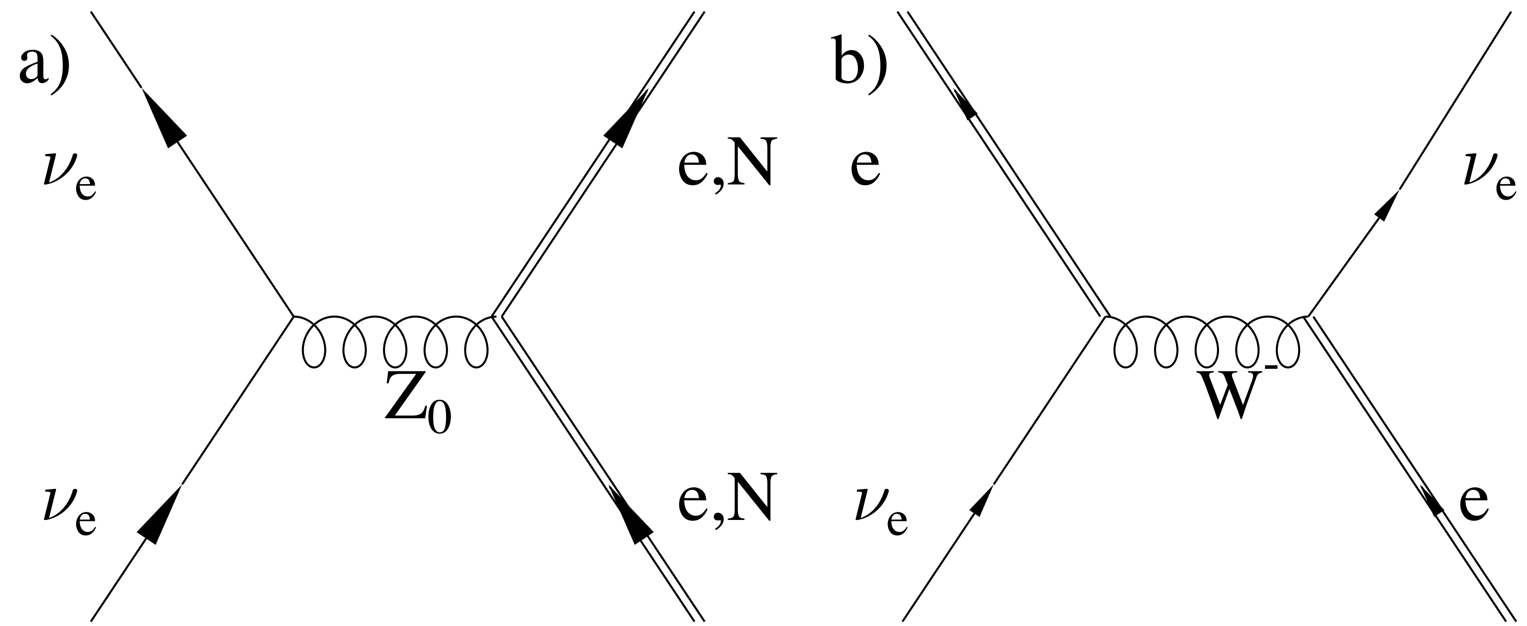
yielding

$$i\frac{d}{dx}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -\delta m^2 \cos 2\theta & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & \delta m^2 \cos 2\theta \end{pmatrix}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix}$$

vacuum m_ν^2 matrix in the flavor basis

where x is the same as t (x = ct). We will assume the mixing angle is small, so that $\nu_e \sim \nu_L, \quad \nu_\mu \sim \nu_H$

solar matter generates a flavor asymmetry



- modifies forward scattering amplitude
- explicitly dependent on solar electron density
- makes the electron neutrino heavier at high density

$$m_{\nu_e}^2 = 4E\sqrt{2}G_F\ \rho_e(x)$$

inserting this into mass matrix generates the 2-flavor MSW equation

$$i\frac{d}{dx}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -\delta m^2 \cos 2\theta + 4E\sqrt{2}G_F\rho_e(x) & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & \delta m^2 \cos 2\theta \end{pmatrix}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix}$$

or equivalently

$$i\frac{d}{dx}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -\delta m^2 \cos 2\theta + 2E\sqrt{2}G_F\rho_e(x) & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & -2E\sqrt{2}G_F\rho_e(x) + \delta m^2 \cos 2\theta \end{pmatrix}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix}$$

the m_ν^2 matrix's diagonal elements vanish at a critical density

$$\rho_c : \quad \delta m^2 \cos 2\theta \equiv 2E\sqrt{2}G_F\rho_c$$

Now suppose we diagonalize the matrix on the right at each x, defining the "local mass eigenstates," and transform to this (evolving) basis inserting this into mass matrix generates the 2-flavor MSW equation

$$i\frac{d}{dx}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -\delta m^2 \cos 2\theta + 4E\sqrt{2}G_F\rho_e(x) & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & \delta m^2 \cos 2\theta \end{pmatrix}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix}$$

or equivalently

U(x) U(x) $U^{-1}(x)U(x)$

$$i\frac{d}{dx}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -\delta m^2 \cos 2\theta + 2E\sqrt{2}G_F\rho_e(x) & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & -2E\sqrt{2}G_F\rho_e(x) + \delta m^2 \cos 2\theta \end{pmatrix}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix}$$

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Now suppose we diagonalize the matrix on the right at each x, defining the "local mass eigenstates," and transform to this (evolving) basis

In terms of these local mass eigenstates

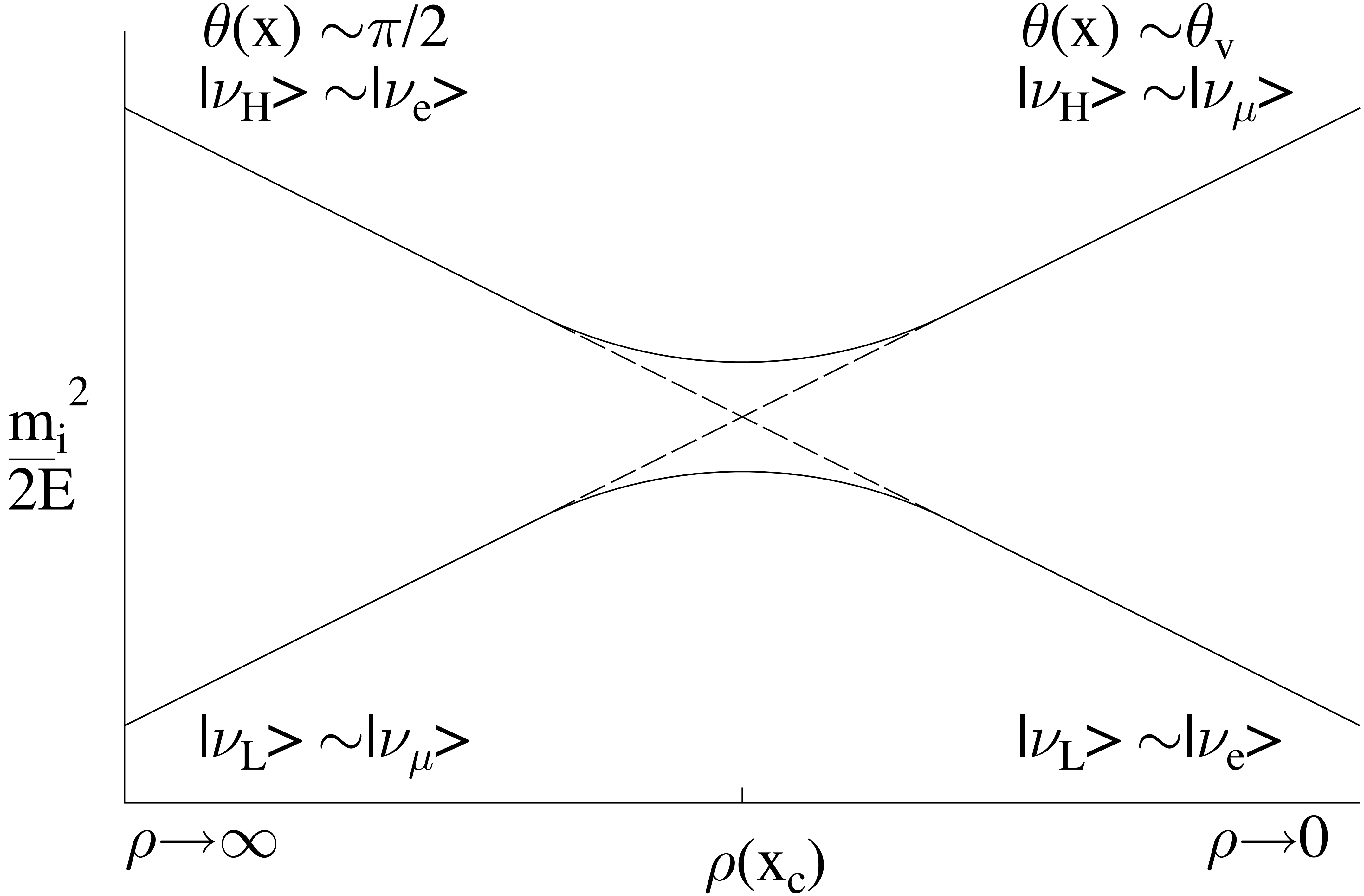
$$|\nu(x)\rangle = a_H(x)|\nu_H(x)\rangle + a_L(x)|\nu_L(x)\rangle$$

$$i\frac{d}{dx}\begin{pmatrix} a_H(x) \\ a_L(x) \end{pmatrix} = \frac{1}{4E}\begin{bmatrix} m_H^2(x) & i\alpha(x) \\ -i\alpha(x) & m_L^2(x) \end{bmatrix}\begin{pmatrix} a_H(x) \\ a_L(x) \end{pmatrix}$$

observe:

- mass splittings small at ρc: avoided level crossing
- $\nu_H(x) \sim \nu_e$ at high density
- if vacuum θ small, $\nu_H(0) \sim \nu_\mu$ in vacuum

thus there is a local mixing angle θ(x) that rotates from $\sim \pi/2 \rightarrow \theta_v$
as $\rho_e(x)$ goes from $\infty \rightarrow 0$



θ(x) ~π/2
|ν_H> ~|ν_e>
θ(x) ~θ_v
|ν_H> ~|ν_μ>
m_i^2 / 2E
|ν_L> ~|ν_μ>
|ν_L> ~|ν_e>
ρ→∞
ρ(x_c)
ρ→0

- it must be that $\alpha(x) \sim \frac{d\rho}{dx}$
- if derivative gentle (change in density small over one local oscillation length) we can ignore: matrix then diagonal, easy to integrate

$$\Rightarrow \; P_{\nu_e}^{adiabatic} = \frac{1}{2} + \frac{1}{2}\cos 2\theta_v \cos 2\theta_i \rightarrow 0 \text{ if } \theta_v \sim 0, \theta_i \sim \pi/2$$

- most adiabatic behavior is near the crossing point: small splitting ⇒ large local oscillation length ⇒ can "see" density gradient

- derivative at ρ_c governs nonadiabatic behavior (Landau Zener)

$$P_{\nu_e}^{LZ} = \frac{1}{2} + \frac{1}{2}\cos 2\theta_v \cos 2\theta_i (1 - 2P_{hop})$$

so $\rightarrow 1 \text{ if } \theta_v \sim 0, \theta_i \sim \pi/2, P_{hop} \sim 1$

$$P_{hop}^{linear} = e^{-\pi\gamma_c/2} \qquad \gamma_c = \frac{\sin^2 2\theta}{\cos 2\theta}\frac{\delta m^2}{2E}\frac{1}{\left|\frac{1}{\rho_c}\frac{d\rho}{dx}\right|} \sim \frac{\text{solar density scale height}}{\text{local oscillation length}}$$

$\Upsilon_c >> 1 \Leftrightarrow$ adiabatic, so strong flavor conversion

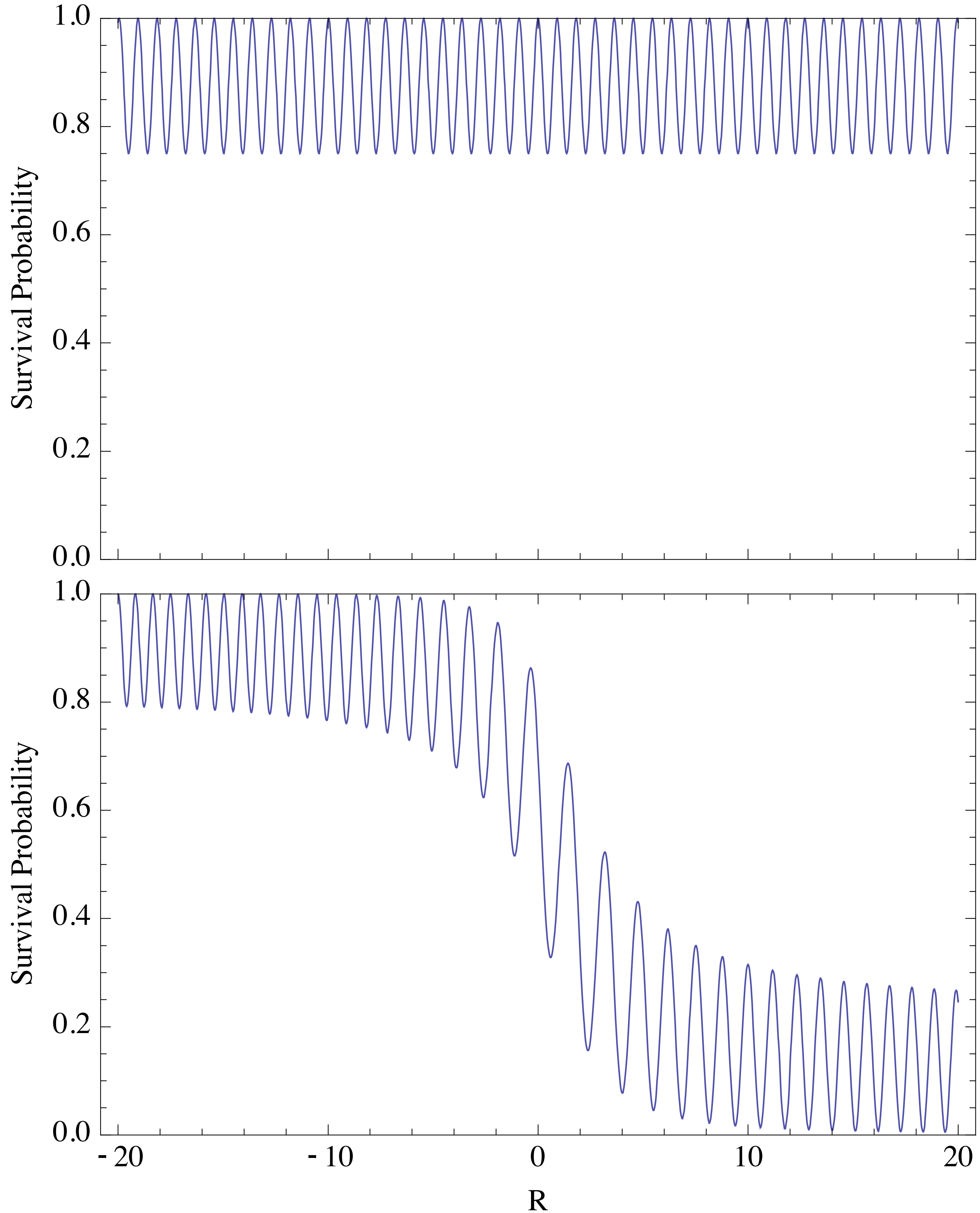
$\Upsilon_c << 1 \Leftrightarrow$ nonadiabatic, little flavor conversion

so two conditions for strong flavor conversion:

sufficient density to create a level crossing

adiabatic crossing of that critical density

MSW mechanism is about passing through a level crossing



Mathematica HW problem

a) vacuum oscillations θ=15°
R from -20 to +20

R in units of $\dfrac{4E\cos 2\theta}{\delta m^2 \sin^2 2\theta}$

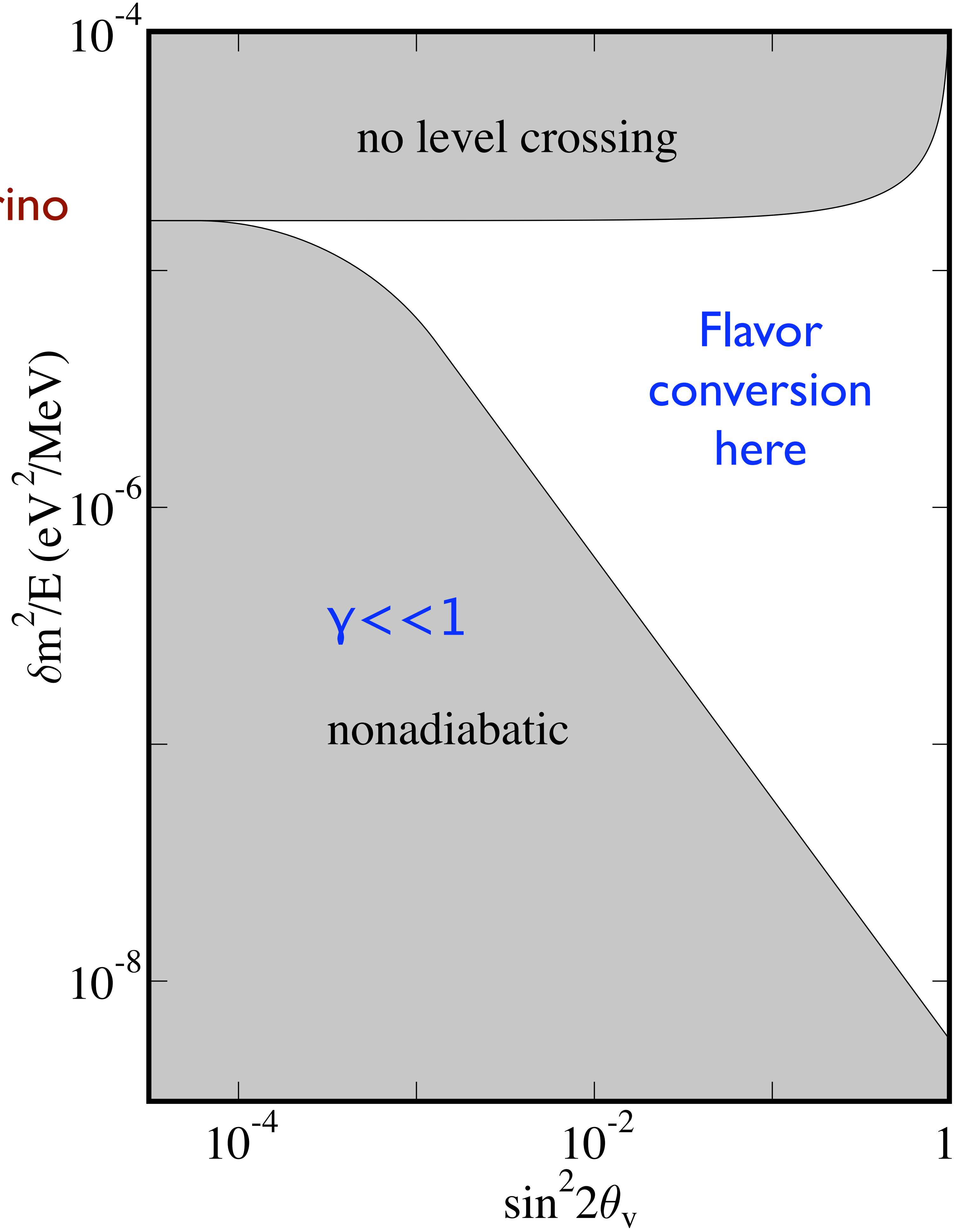
b) matter oscillations

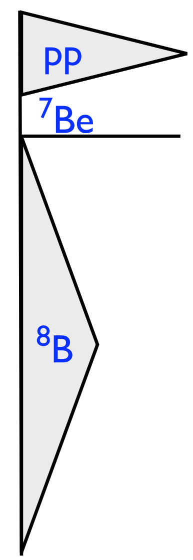
add $\rho_e(R) \propto 1 - \dfrac{2}{\pi}\arctan aR$

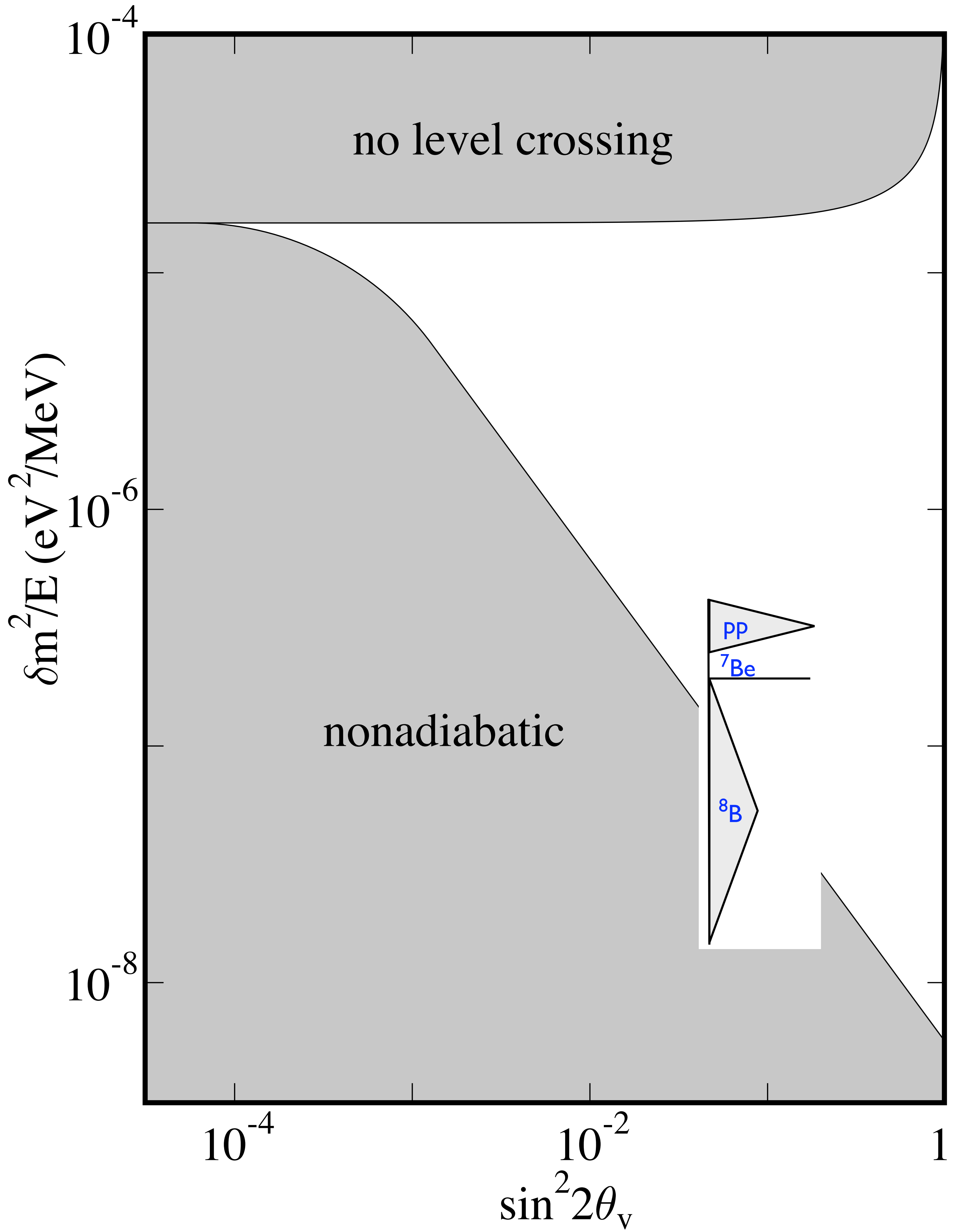
normalize so that crossing occurs at R = 0

note $\rho_e(R) \to 0$ as $R \to \infty$

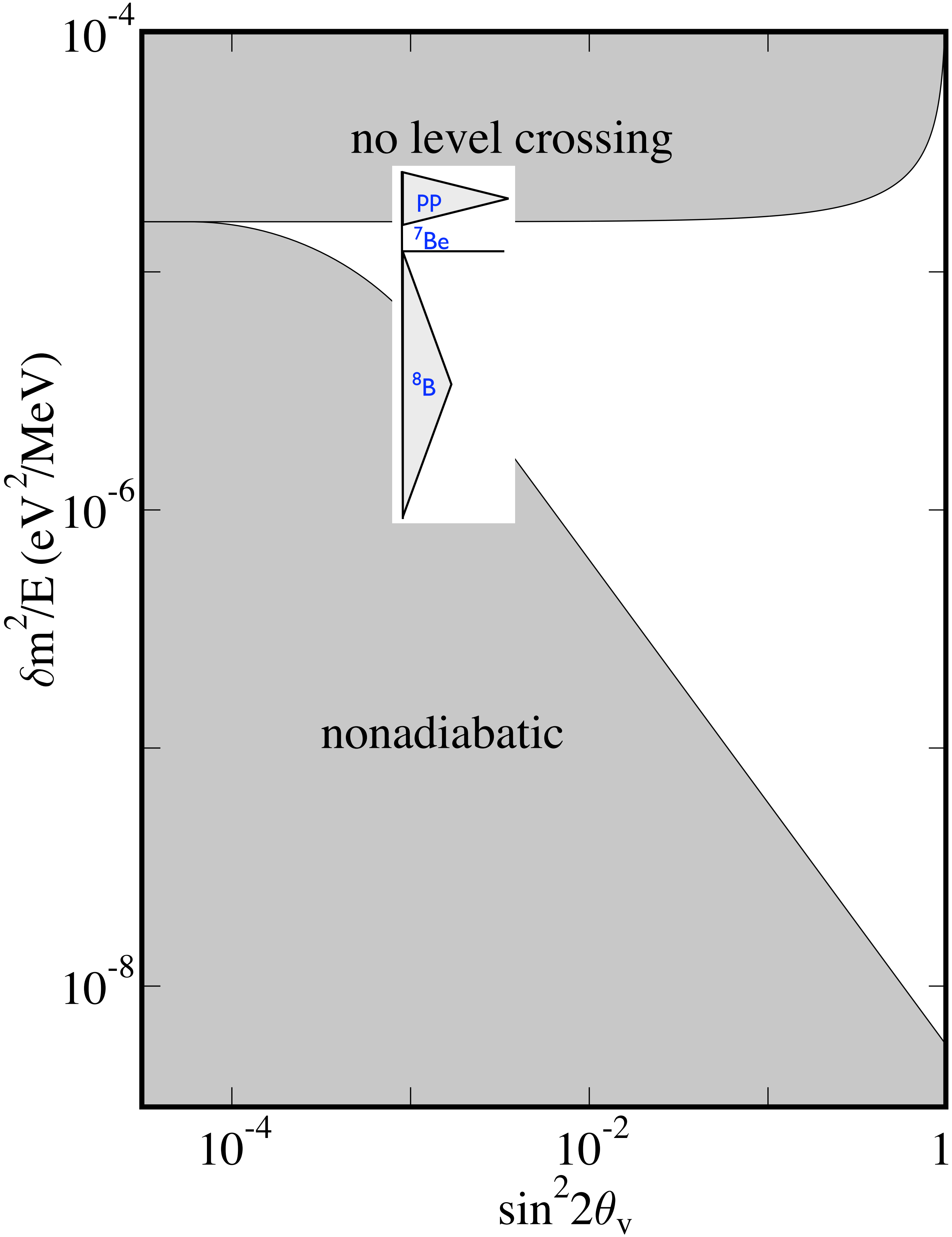
So ν_e is produced as a heavy eigenstate, then propagates toward the vacuum, where it is the light eigenstate solving the solar neutrino problem



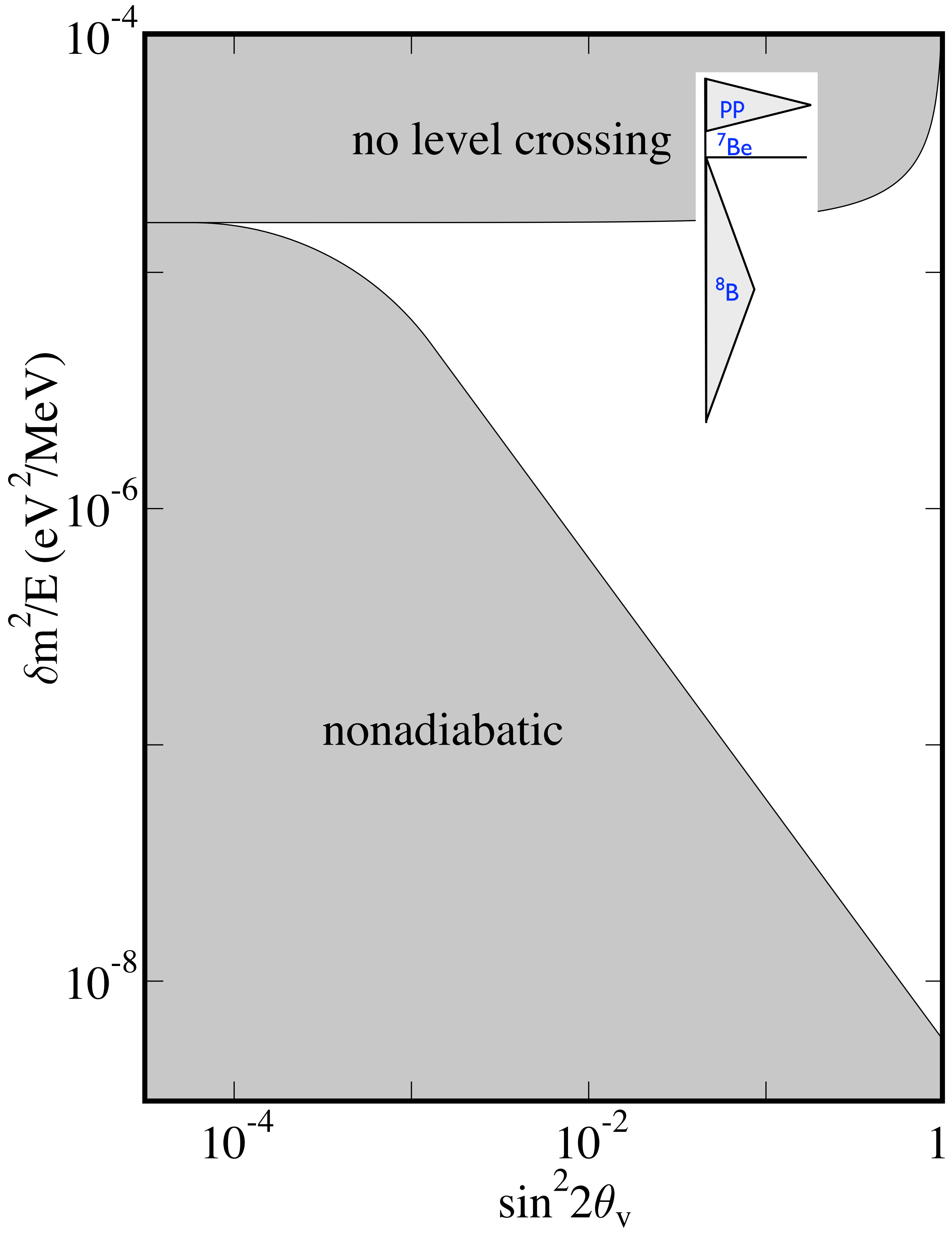




Low solution



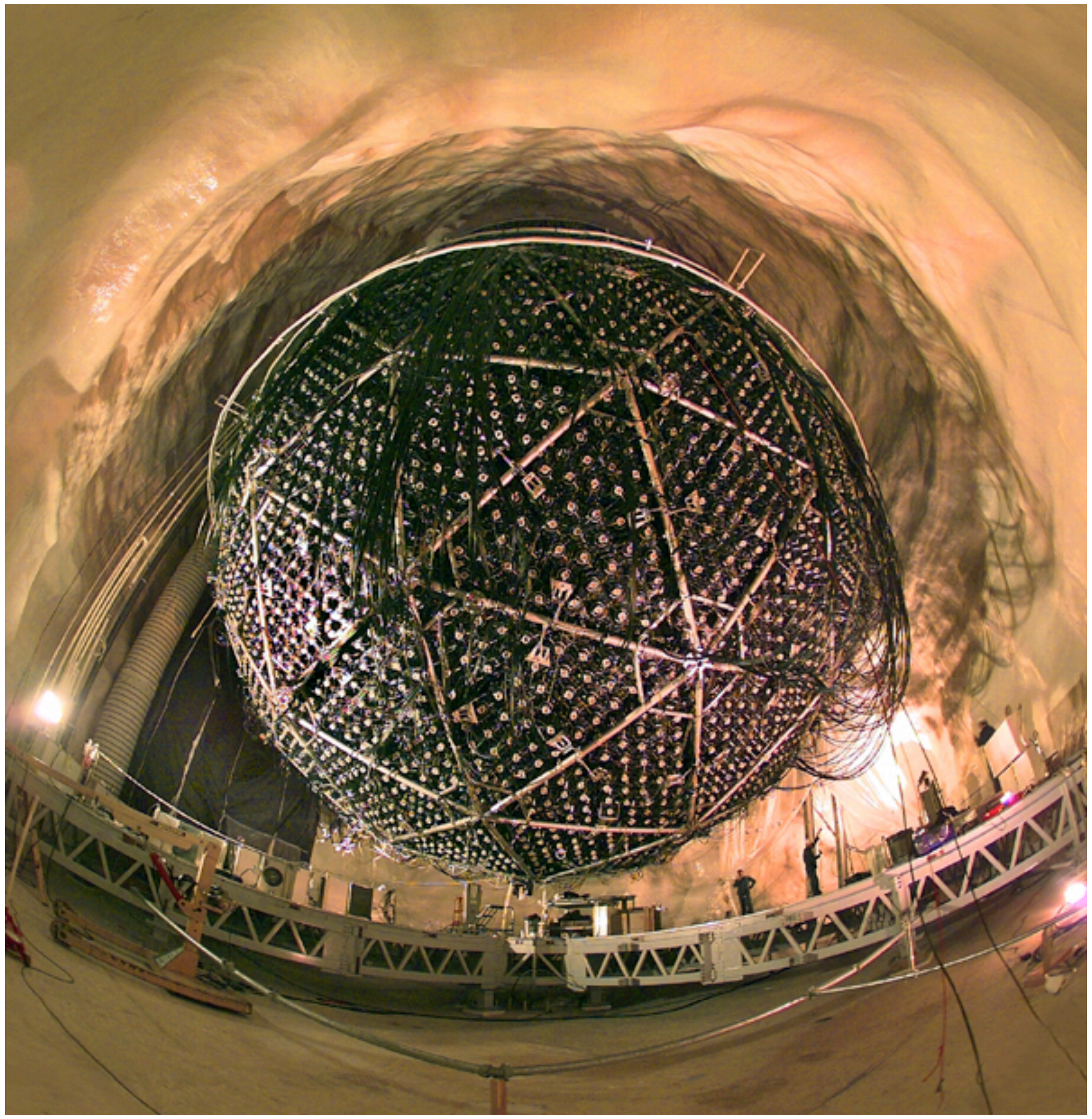
Small angle solution

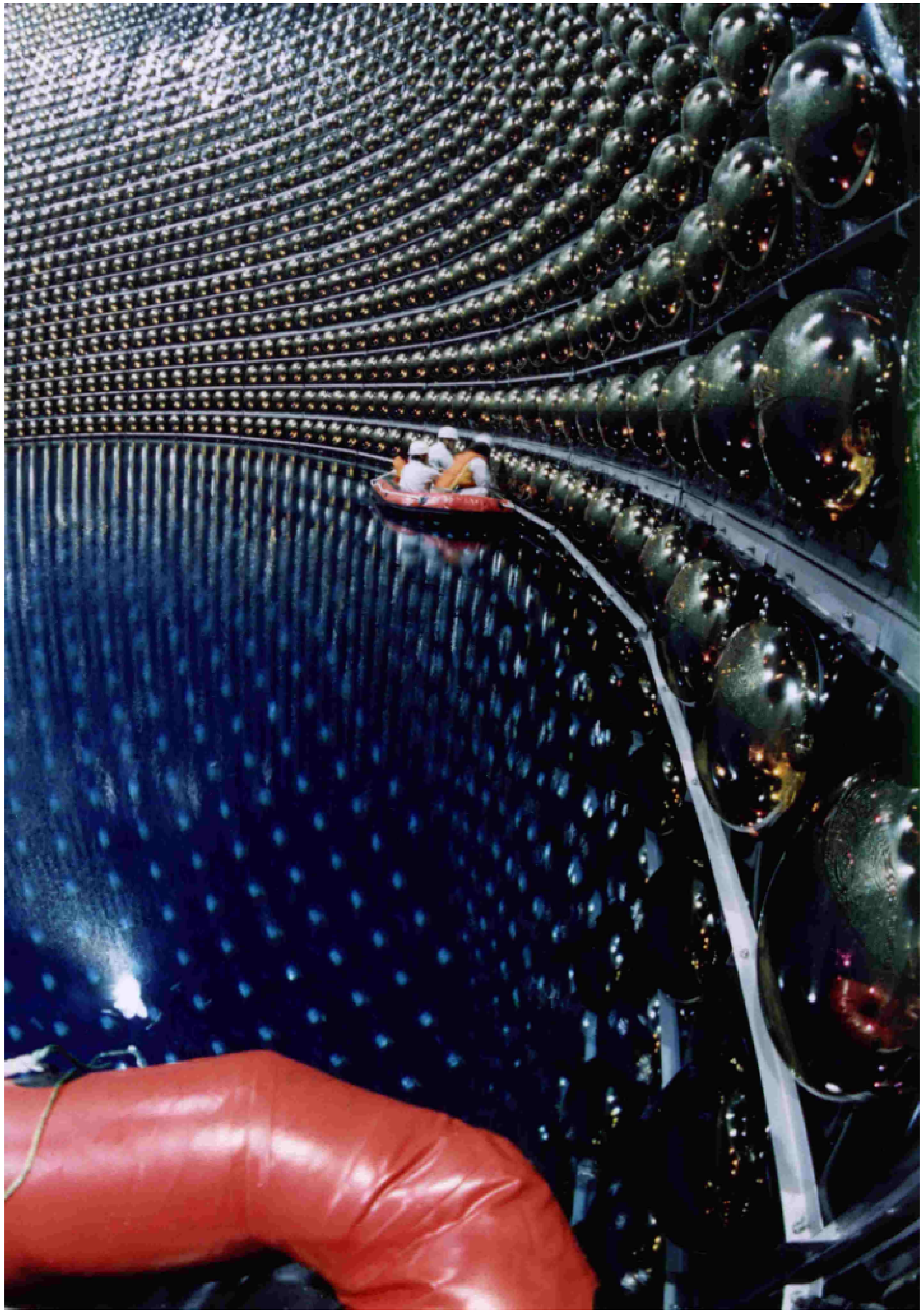


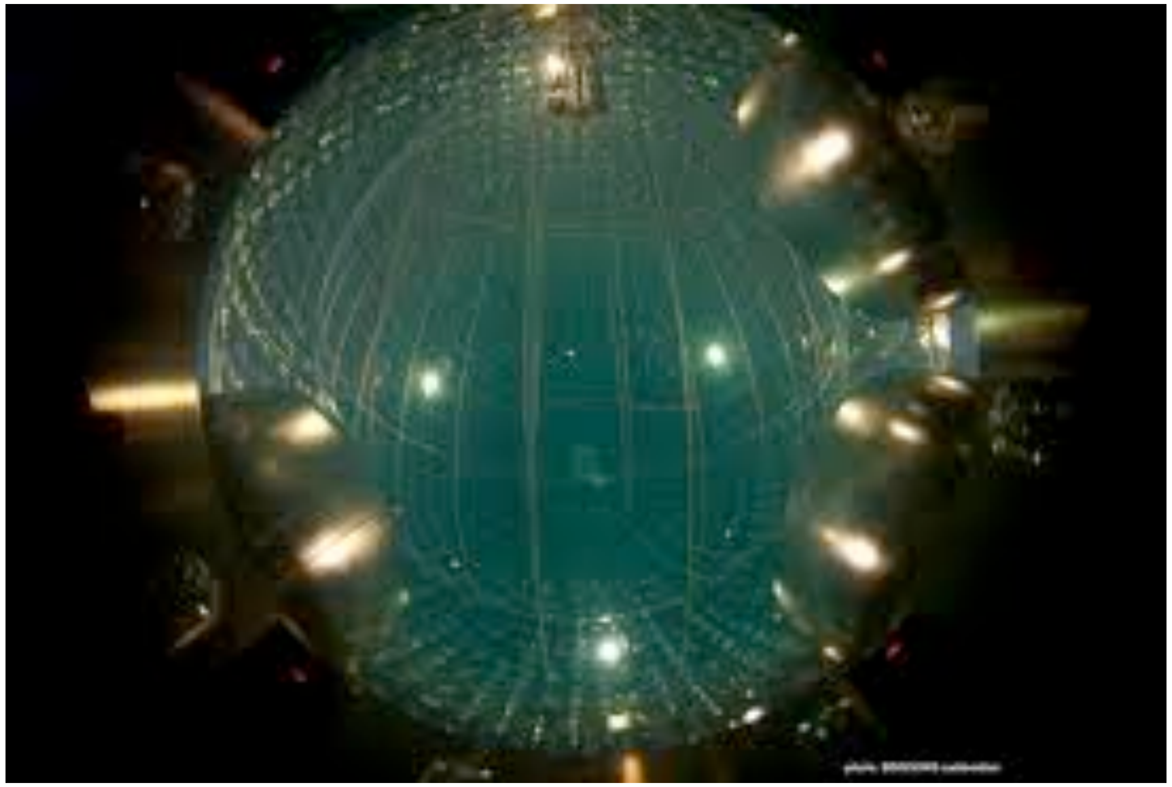
Large angle solution

this is the solution matching SNO and SuperK results + Ga/Cl/KII

$\tan^2\theta_v \sim 0.40$







SNO, Super-Kamiokande, Borexino

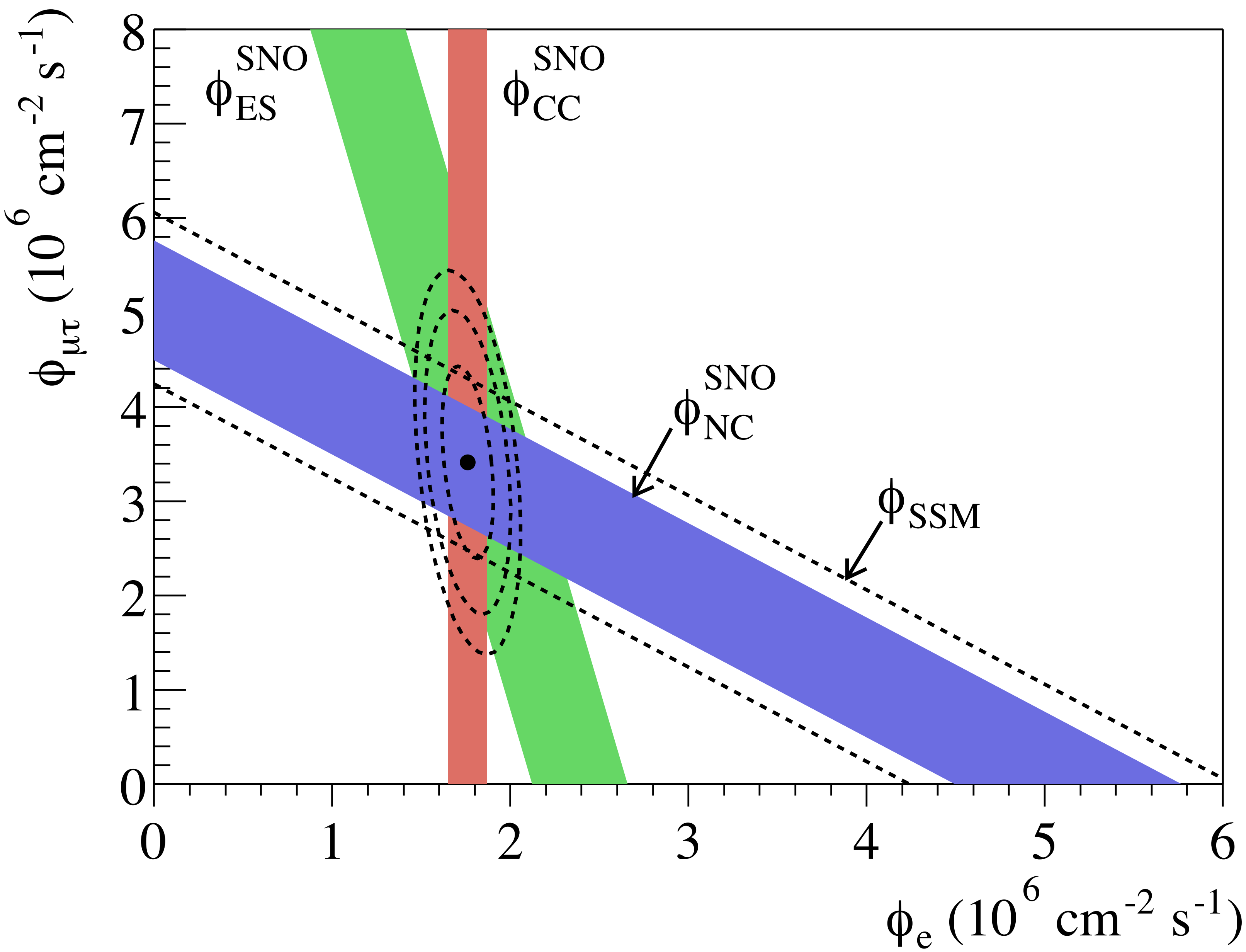
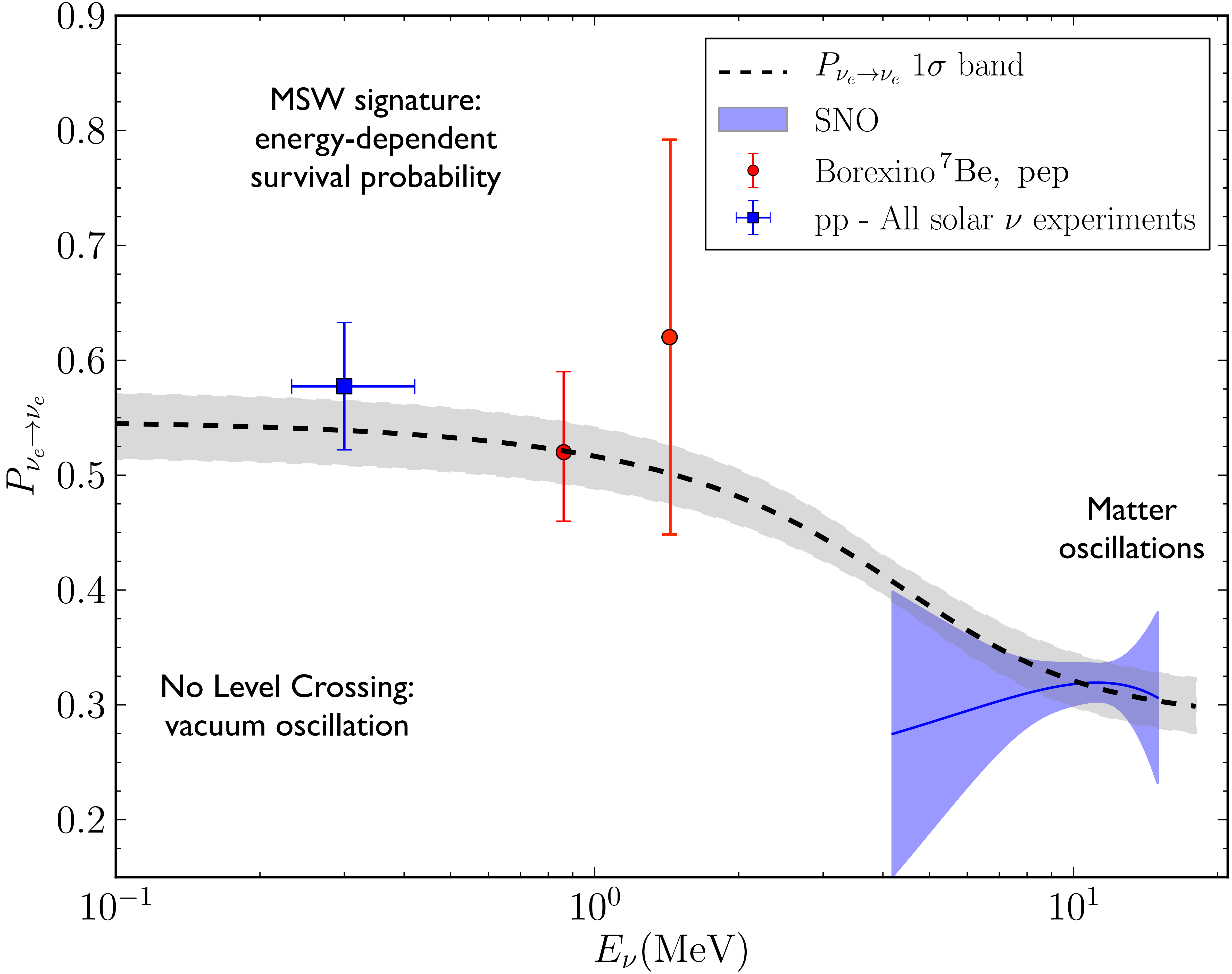


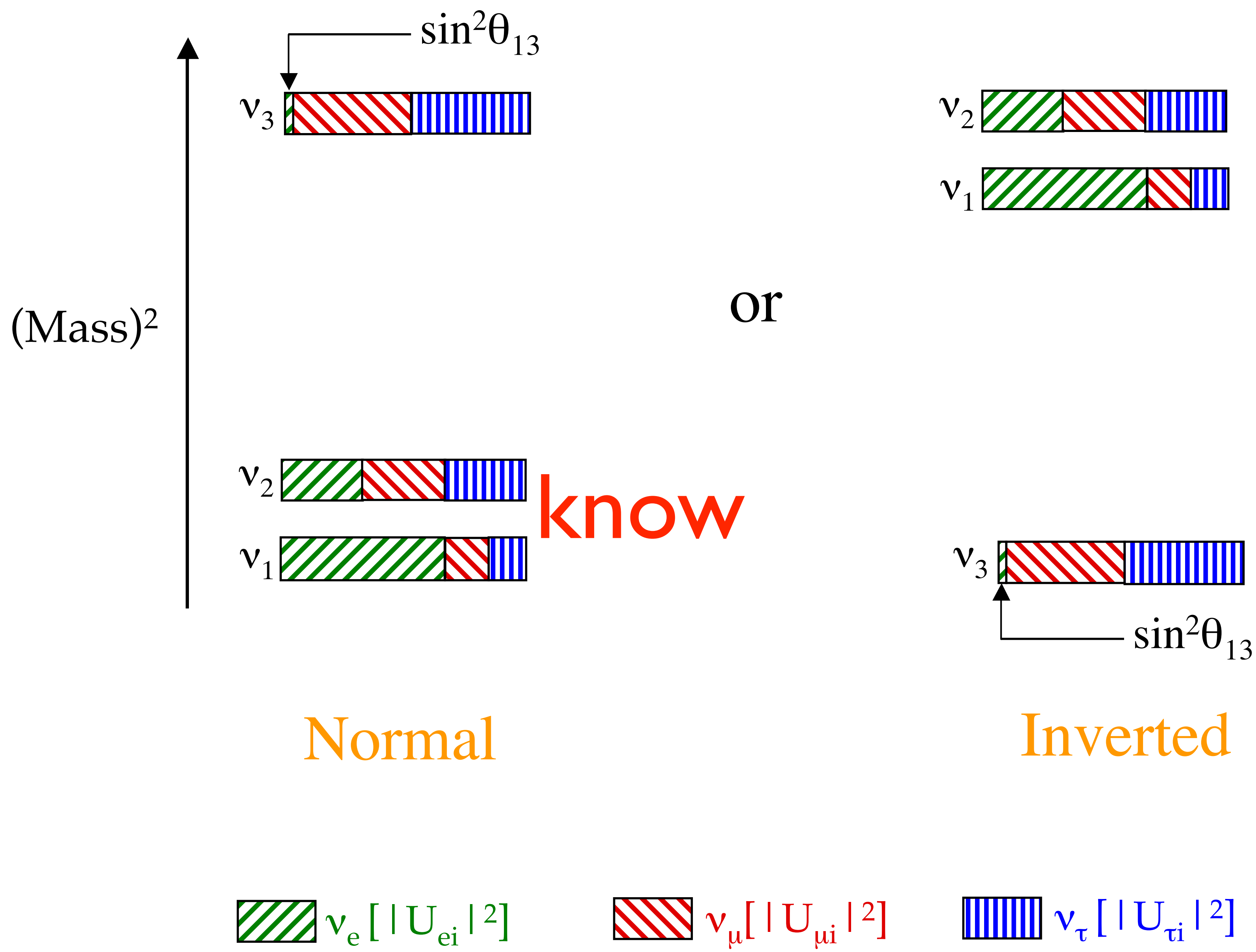
Figure 2: Flux of ^{8}B solar neutrinos is divided into ν_μ/ν_τ and ν_e flavors by the SNO analysis. The diagonal bands show the total ^{8}B flux as predicted by the SSM (dashed lines) and that measured with the NC reaction in SNO (solid band). The widths of these bands represent the $\pm 1\sigma$ errors. The bands intersect in a single region for $\phi(\nu_e)$ and $\phi(\nu_\mu/\nu_\tau)$, indicating that the combined flux results are consistent with neutrino flavor transformation assuming no distortion in the ^{8}B neutrino energy spectrum.



0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
$P_{\nu_e \to \nu_e}$
10^{-1}
10^{0}
10^{1}
E_ν(MeV)
$P_{\nu_e \to \nu_e}$ 1σ band
SNO
Borexino ^{7}Be, pep
pp - All solar ν experiments
MSW signature: energy-dependent survival probability
No Level Crossing: vacuum oscillation
Matter oscillations

Our results to date on neutrino properties

Hierarchy



$$\delta m^2_{31} = \begin{cases} (2.47^{0.06}_{-0.10}) \times 10^{-3}\text{eV}^2, & \text{NH} \\ -(2.37^{+0.07}_{0.11}) \times 10^{-3}\text{eV}^2, & \text{IH} \end{cases}$$

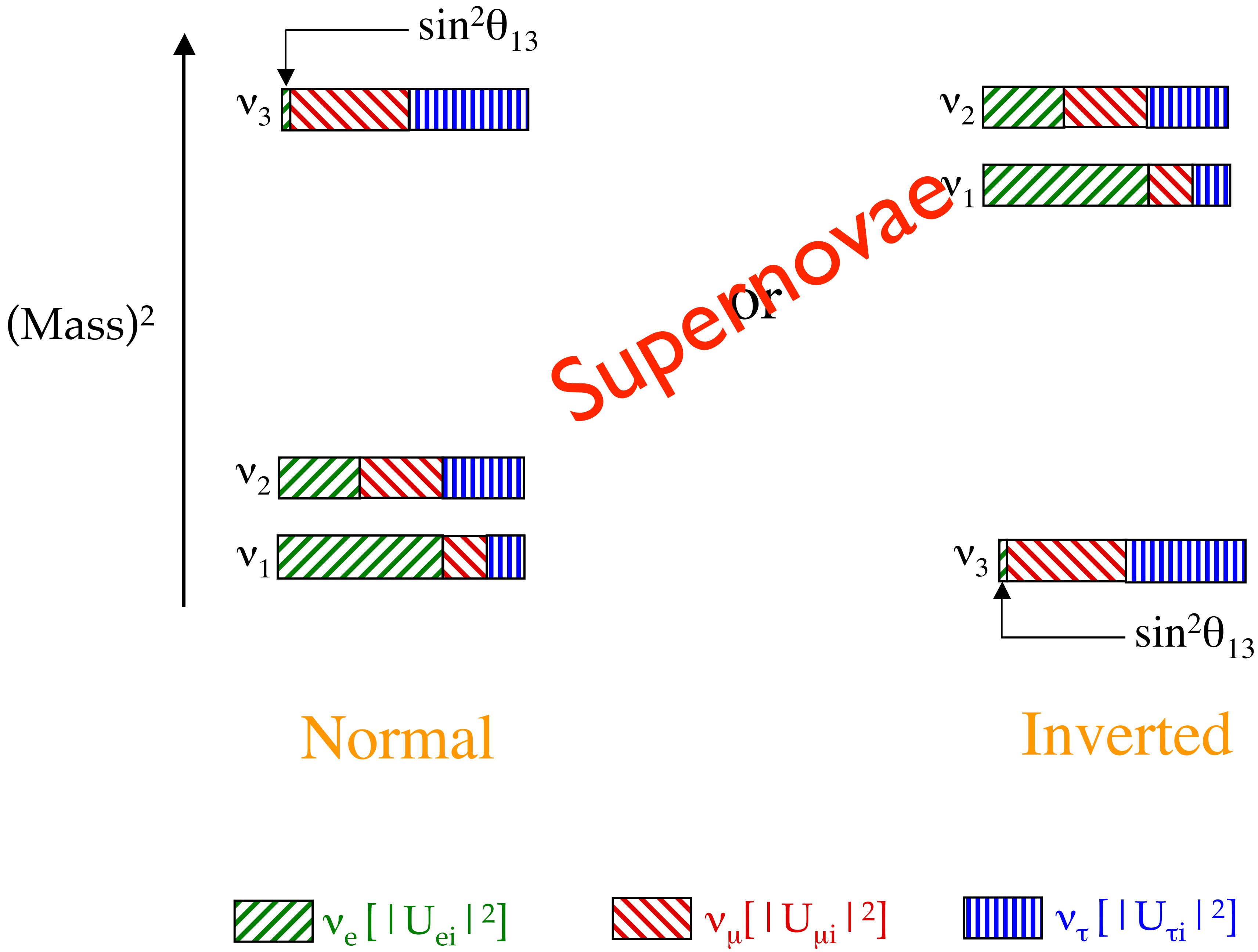
$$\delta m^2_{21} = 7.54^{+0.26}_{-0.22} \times 10^{-5}\text{eV}^2$$

Bari global analysis
(Valencia quite similar)

Δ_{12} $|\Delta_{23}|$ $\text{sign}[\Delta_{23}]$ absolute scale

Our initial results on neutrino properties

Hierarchy



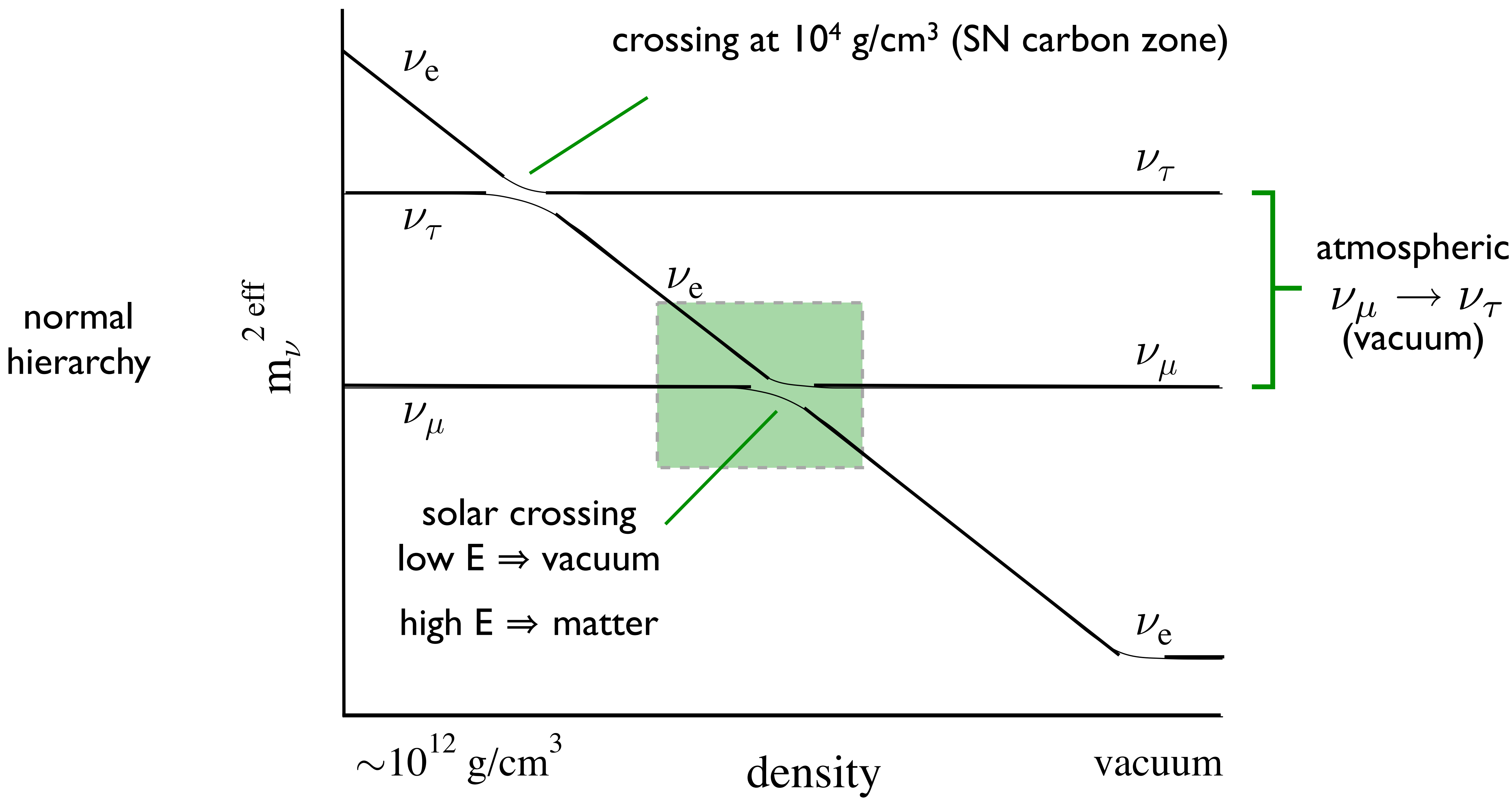
$$\delta m^2_{31} = \begin{cases} (2.47^{0.06}_{-0.10}) \times 10^{-3}\mathrm{eV}^2, & \mathrm{NH} \\ -\,(2.37^{+0.07}_{0.11}) \times 10^{-3}\mathrm{eV}^2, & \mathrm{IH} \end{cases}$$

$$\delta m^2_{21} = 7.54^{+0.26}_{-0.22} \times 10^{-5}\mathrm{eV}^2$$

Bari global analysis
(Valencia quite similar)

Δ_{12} $|\Delta_{23}|$ $\mathrm{sign}[\Delta_{23}]$ absolute scale

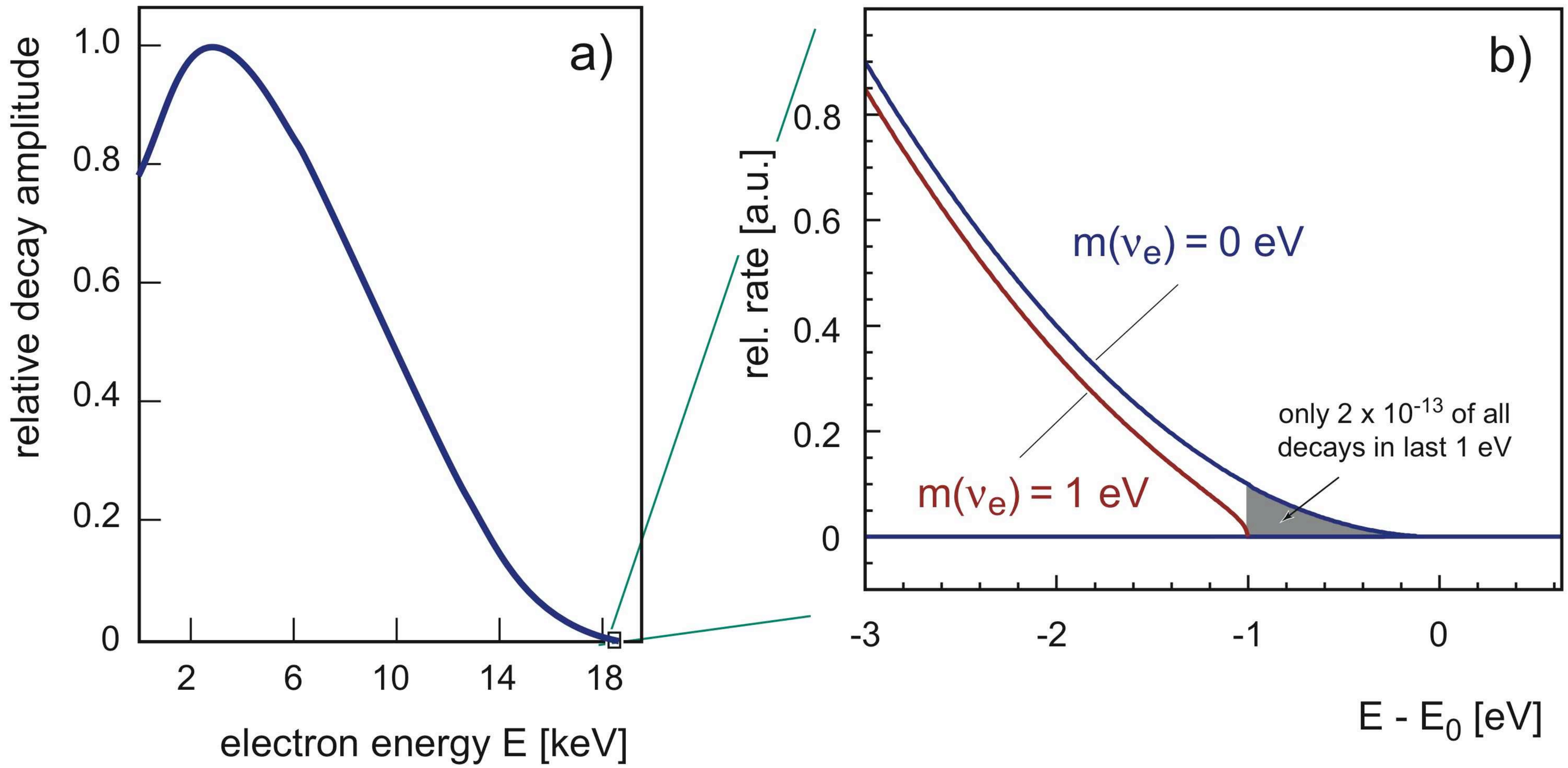
The hierarchy question is one reason supernovae might be interesting



there is an opportunity to explore the second MSW crossing because of the high densities available in the supernovae envelope

Direct (kinematic) tests of neutrino mass

$$m_{\bar{\nu}_e} = \sum_{i=1}^{n} |U_{ei}|^2 m_i$$



Known splittings << resolution: 2.2 eV limit ⇒ $\sum_{i=1}^{n} m_\nu(i) \leq 6.6 \text{ eV}$

It will be hard for laboratory measurements to compete with cosmology

Katrin's goal is 250 meV for m_{ν_e} - not the sum

But many will be uncomfortable with a cosmological claim for neutrino mass -- and possibly also for the hierarchy -- without some direct laboratory confirmation

The mixing

$$\text{undetermined}: \delta; \quad \phi_1, \phi_2$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} \nu_1 \\ e^{i\phi_1}\nu_2 \\ e^{i\phi_2}\nu_3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ e^{i\phi_1}\nu_2 \\ e^{i\phi_2}\nu_3 \end{pmatrix}$$

Atmospheric | Reactor (Daya Bay/Reno/Double Chooz) | Solar

$$\sin^2\theta_{23} = \begin{cases} 0.386^{+0.024}_{-0.021}, & \text{NH} \\ 0.392^{+0.039}_{-0.022} & \text{IH} \end{cases}$$

$$\Rightarrow (3\sigma) \begin{cases} 0.331 \leftrightarrow 0.637, & \text{NH} \\ 0.335 \leftrightarrow 0.663 & \text{IH} \end{cases}$$

$$\sin^2\theta_{13} = \begin{cases} 0.0241 \pm 0.0025, & \text{NH} \\ 0.0244^{+0.0023}_{-0.0025}, & \text{IH} \end{cases}$$

$$\sin^2\theta_{12} = 0.307^{+0.018}_{-0.016}$$

Bari global analysis
(Valencia quite similar)

The knowledge of the mixing angles gained from oscillations helps define the possibilities for double beta decay, despite our ignorance of both the hierarchy and absolute mass scale

Consider double beta decay with three light Majorana neutrinos

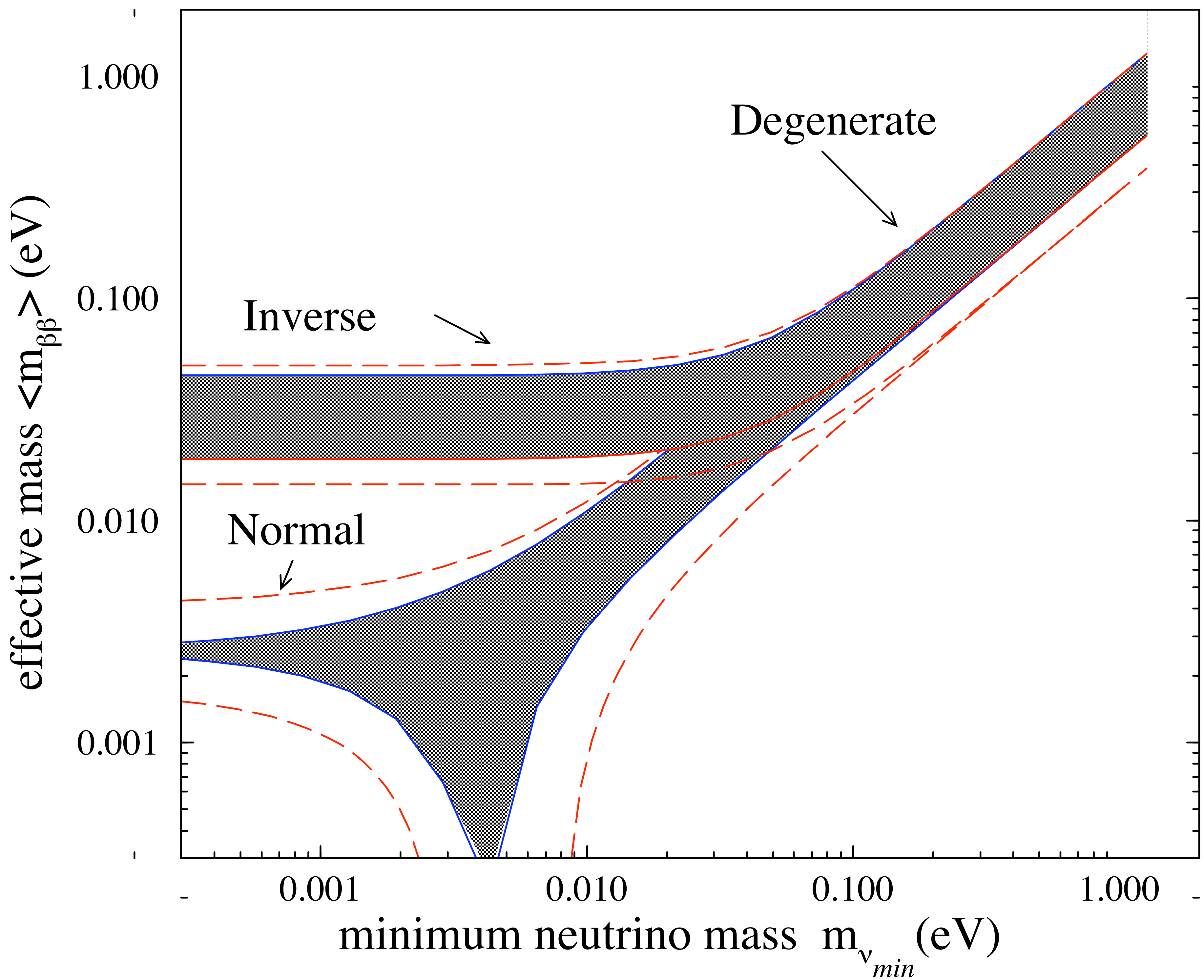
$$|\langle m_{\beta\beta}\rangle| \to \left| \sum_{i=1,\text{light}}^{N} \lambda_i^{c*}\, U_{ei}^L U_{ei}^L\, m_\nu^i \right| \sim \left| m_\nu^1 |U_{e1}|^2 + m_\nu^2 |U_{e2}|^2 e^{2i\phi_1} + m_\nu^3 |U_{e3}|^2 e^{2i(\phi_2-\delta)} \right|$$

$$\sim \left| c_{12}^2 c_{13}^2 m_\nu^1 + m_\nu^2 s_{12}^2 c_{13}^2 e^{2i\phi_1} + m_\nu^3 s_{13}^2 e^{2i(\phi_2-\delta)} \right|$$

NH: $\langle m_{\beta\beta}\rangle \sim \left| \sqrt{\delta m_{21}^2}\; s_{12}^2 c_{13}^2 + \sqrt{|\delta m_{31}^2|}\; s_{13}^2 e^{i\phi} \right| \sim |4.8 + 1.2 e^{i\phi}|\ \text{meV}$

IH: $\langle m_{\beta\beta}\rangle \sim \sqrt{|\delta m_{31}^2|}\; c_{13}^2\; \sqrt{1 - \sin^2 2\theta_{12} \sin^2\phi} = [19 \leftrightarrow 49]\ \text{meV}$

degenerate: $\langle m_{\beta\beta}\rangle \sim m_0 \left| c_{12}^2 c_{13}^2 e^{i\phi} + s_{12}^2 c_{13}^2 e^{i\phi'} + s_{13}^2 \right| \sim m_0 (0.68 \pm 0.32)$



1.000
0.100
0.010
0.001
effective mass <m_ββ> (eV)
Degenerate
Inverse
Normal
0.001
0.010
0.100
1.000
minimum neutrino mass m_ν_min (eV)

CP-violation

- Dirac CP-violation phase δ measurable in flavor oscillations
- signal would be an asymmetry in $P(\nu_\mu \to \nu_e)$ vs. $P(\bar{\nu}_\mu \to \bar{\nu}_e)$
- practical long-baseline experiments typically involve baselines of 1000 to 3000 km
  - matter effects! a clean experiment would require earth and anti-earth comparisons (another set of parameter degeneracies)
- CPNC invariant is

$$\begin{aligned} J_{CP} &= \sin\theta_{12}\sin\theta_{23}\sin\theta_{13}\cos\theta_{12}\cos\theta_{23}\cos^2\theta_{13}\sin\delta \\ &\sim 0.2\sin\theta_{13}\sin\delta \end{aligned}$$

- so could be as large as $\sim 0.04\ \sin\delta$ depending on θ_{13}
- can compare to analogous CKM quantity $J_{CP}^{CKM} \sim 3 \times 10^{-5}$

- appearance signals for $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ or $P(\nu_\mu \rightarrow \nu_e)$ vary as

$$\sim \frac{1}{2}\sin^2 2\theta_{13}\sin^2\left(\frac{\delta m^2_{31} L}{4E_\nu}\right) + \left(\begin{array}{c}\text{matter}\\ \text{effects}\end{array}\right) \pm 4J_{CP}\sin^2\left(\frac{\delta m^2_{31} L}{4E_\nu}\right)\sin\left(\frac{\delta m^2_{21} L}{2E_\nu}\right) + ...$$

- for typical proton drivers producing 1-3 GeV ν beams, $L(\delta m_{31}) \sim 700 \text{ km}$

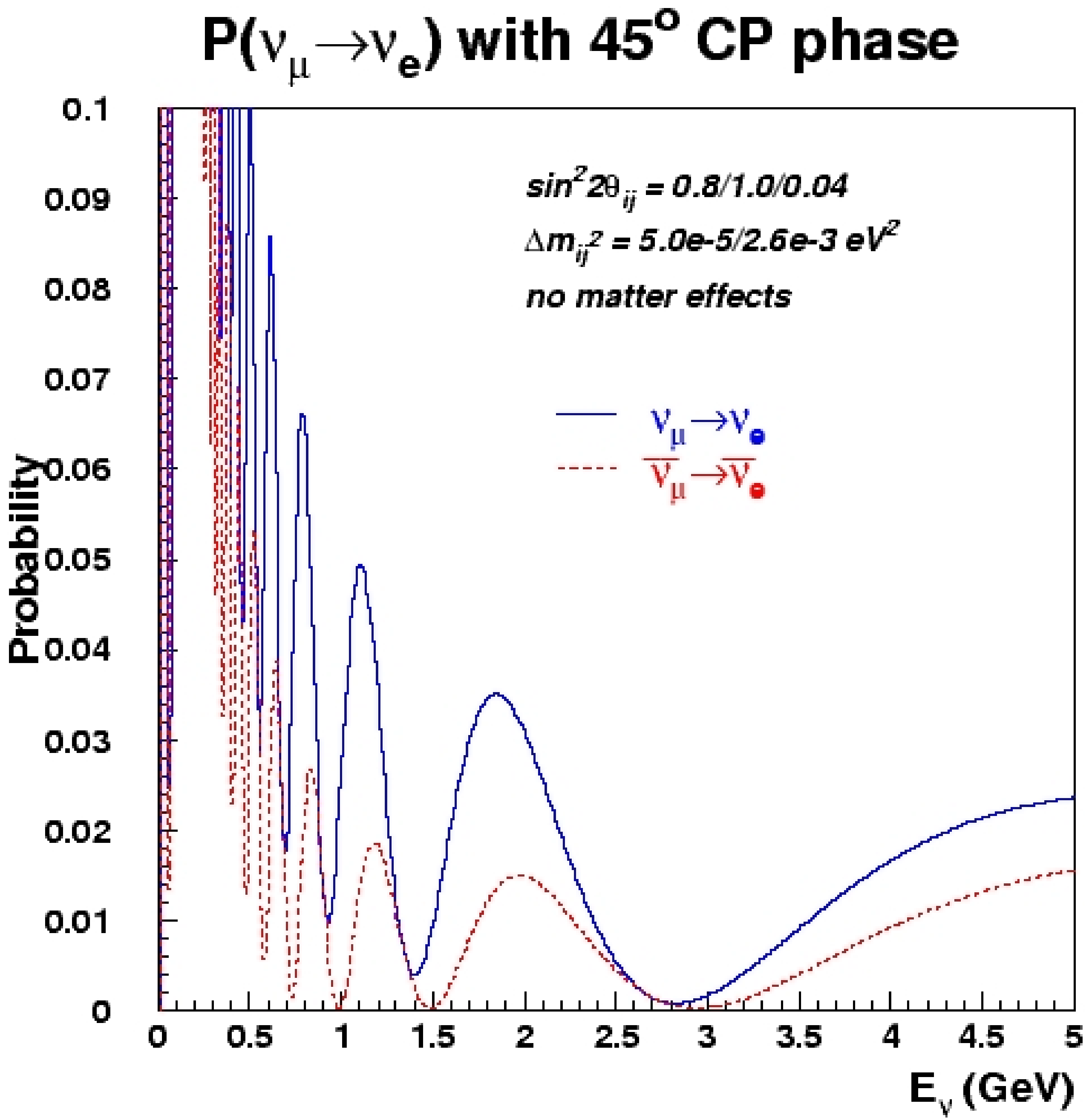
- CPNC term controlled by smaller δm^2_{21} $\Rightarrow$ grows linearly

$$\sin\left(\frac{\delta m^2_{21} L}{2E_\nu}\right) \sim L/E\nu$$

- so signal grows but flux drops as $1/L^2$, so signal/background can degrade: complicated optimization that tens to give $L \sim$ 1500-3000 km

- one strategy employs a broad beam, with several oscillation minima imprinted on the spectrum -- helps disentangle the various effects

- requires a detector is the 0.1-0.5 megaton range



from BNL study
Marciano et al.

The known unknowns. We do not know

- the absolute scale of ν mass
- the Dirac/Majorana nature of the mass
- the hierarchy, normal or inverted
- the sizes or roles in nature of three CP-violating phases
- we have not explored other (nonsolar) MSW crossings or potentials
- we do not know whether νs have nonzero electromagnetic moments
- we do not know whether there are additional ν species
- we do not know whether the universe is lepton number asymmetric

and we have left this hard stuff for you young researchers to resolve